INTRODUCTION TO GKM THEORY

SHINTARÔ KUROKI

Let T be an n-dimensional compact torus, i.e., the n-fold product of the circle group S^1, $T \simeq S^1 \times \cdots \times S^1 =: (S^1)^n$. In 1999, Goresky, Kottwiz, and MacPherson established a framework for studying the class of manifolds with a T-action, known as *equivariantly formal manifolds*, by using their fixed points and one-dimensional orbits. These manifolds are now commonly referred to as *GKM manifolds*. Expanding on their work, Guillemin and Zara introduced the notion of an *abstract GKM graph* in 2001 as a combinatorial counterpart of GKM manifolds, thus initiating the study of spaces with T-actions using the combinatorial structure of GKM graphs. Since then, the research of GKM manifolds and GKM graphs, commonly known as *GKM theory*, has been the subject of extensive research.

The class of GKM manifolds includes the wide class of T-manifolds, such as (quasi)toric manifolds, homogeneous space G/H, where G is a compact, connected Lie group and H is its closed subgroup with the same maximal torus in G, and regular semisimple Hessenberg varieties $\mathrm{Hess}(A, h)$ etc.

In these five talks, I will introduce GKM theory, starting with the basics of equivariant cohomology. The goal of these talks is to provide the tools to compute the graph equivariant cohomology of a GKM graph (or the equivariant cohomology of certain GKM manifolds).

The topics covered will include:

(1) Basics of equivariant cohomology
(2) GKM manifolds and their GKM graphs
(3) Abstract GKM graphs and their graph equivariant cohomology
(4) The Chang-Skjelbred Lemma: When the equivariant cohomology of a GKM manifold is isomorphic to the graph equivariant cohomology of its GKM graph
(5) Explicit computations of graph equivariant cohomology

Okayama University of Science
Email address: kuroki@ous.ac.jp