Single Event Transition Risk (SETR): Carbon-transition Premium to Carbon-transition Risk

Professor Siddhartha Pratim Chakrabarty
Department of Mathematics
Indian Institute of Technology Guwahati
(Joint work with: Suryadeepto Nag (University of Lausanne) and Sankarshan Basu (IIM Bangalore))

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3. This fundamental paradigm shift is expected to trigger an element of considerable risk, for investors of firms bearing high carbon footprint.
4. The narrative on climate-related factors, the resulting financial risk, and its consequent management is critical.
5. Prudent measurement and transparent disclosure of risk exposures are the primary drivers of this narrative.

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5. Increasing calls for voluntary reporting to be replaced with a formal regulatory framework.

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3. Narrative on transition risk premium has two two perspective: Overall transition risk and specific risk components.
4. Higher carbon footprint resulted in higher returns.
5. The transition risk was short-term (long-term) for countries more dependent on traditional fossil fuel (with stricter regulations).

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4. Recognition that any investment in fossil-fuel industries today will continue to pose carbon risk for the succeeding three to four decades.

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Carbon Transition Risk Premium

1. The question is whether investors are aware of this risk, in which case, the investors, to stay invested, will expect an incentive, like higher returns on the short term, based on the amount of carbon emissions. This is called the "Carbon-Transition Premium" or "Carbon-Risk Premium".

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2. The carbon risk premium $p_{i,T}$, for firm i, in the year T is defined as,

$$p_{i,T} = \rho_{i,T} - \rho^0_{i,T},$$

where $\rho_{i,T}$ and $\rho^0_{i,T}$ are the total annual stock returns, and the risk-free stock returns, in the year T.

Carbon Transition Risk Premium

In order to estimate the existence of such a premium (if any), we perform a cross-sectional analysis with the following linear regression model:

$$r_{i,T} = \alpha_0 + \alpha_1 \ln\left(1 + \mathcal{E}_{i,T-1}\right) + \alpha_2 \cdot \mathcal{C}_{i,T} + \epsilon, \tag{1}$$

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1. $\mathcal{E}_{i,T-1}$ refers to the total carbon emissions (Scope 1+Scope 2).
2. $\mathcal{C}_{i,T}$ is an 8-dimensional vector of the values of the control variables, namely, log-total capitalization, log-ratio of debt-to-book value, ratio of invested capital-to-book value, ratio of book value-to-total capitalization, average momentum, average historical volatility, log-revenue and log-expenses.

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To be estimated: α_0, α_1 and α_2, with α_1 being the critical parameter, pertaining to the existence and value of the carbon risk premium.

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Therefore, we calculate the premium by subtracting the risk-free annual return $\rho^0_{i,T}$,

$$p_{i,T} = \rho_{i,T} - \rho^0_{i,T} = \rho^0_{i,T} \left(e^{\alpha_1 \ln(1+\mathcal{E}_{i,T-1})} - 1 \right). \qquad (2)$$

Carbon Transition Risk Premium

We obtain the following result of the linear regression.

	α_0	α_1	α_2							
Variables	Constant	$\log(1+\mathcal{E})$	Total-cap	Log debt-to-book	Invested capital-to-book	Book-to-total cap	Momentum	Volatility	Log-Revenue	Log-Expenses
Coefficients	-279.96656	0.00151	0.00628	0.00091	0.00012	-0.00846	280.14561	-11.03386	-0.00914	0.00112
Standard error	2.57892	0.00144	0.00380	0.00286	0.00049	0.00642	2.57739	0.22190	0.00393	0.00211
95% CI -lower limit	-287.41780	-0.00267	-0.00464	-0.00686	-0.00198	-0.02573	272.60374	-11.83447	-0.02060	-0.00448
95% CI -upper limit	-272.42448	0.00562	0.01728	0.01065	0.00143	0.01201	287.58325	-10.42770	0.00198	0.00779

Figure: Estimated Coefficients in Cross Section Analysis

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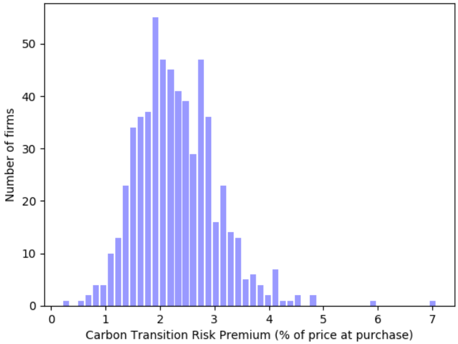
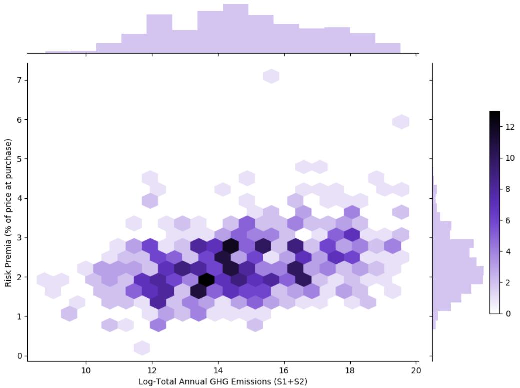
We find a positive value for $\alpha_1 = 0.00151$. This means a greater carbon footprint gives higher returns to investors, *i.e.*, there exists a significant positive carbon premium in the stock prices of the firms in our data set.

Carbon Transition Risk Premium



(a) Histogram of Risk Premia



(b) Variation of Carbon Risk Premium with log-emissions

Single Event Transition Risk

- Since the exact nature of the arrival of the carbon transition risk is impossible to predict, the exact magnitude of the risk at a given time cannot be modelled. Therefore, instead of the exact risk function, we develop a risk measure called the Single Event Transition risk (SETR), that gives us the maximum exposure of risk to a single transition event.

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- We model the arrival of the transition event as a stochastic process. The transition event itself can be modelled by a stochastic risk process $\mathcal{R}(t)$, defined by,

$$\mathcal{R}(t) = \begin{cases} 1, & \text{The transition event occurs at } t, \\ 0, & \text{The transition event does not occur at } t. \end{cases}$$

Single Event Transition Risk

Let $\mathbb{P}[(t_A, t_B)]$ denote the probability of the transition event taking place in an interval (t_A, t_B) *i.e.*, the probability that,

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Hence, for an initial time t_0, such that the low-carbon transition is yet to happen, we can model the arrival process by a probability density function $\tau(t) \in [0, \infty)\ \forall\ t \in (t_0, \infty)$ such that,

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We additionally know that, $\int_{t_0}^{\infty} \tau(t)dt = 1.$

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- We can define a real valued function $P_i(t) : [t_0, \infty) \to \mathbb{R}$ that describes the potential fall of the price of each share of firm i, on any day t, on which the transition event occurs. Suppose that the time at which the transition event occurs is t_e *i.e.*, $\mathcal{R}(t_e) = 1$. Therefore we can model the price of a stock generally as

$$S_i^*(t) = S_i(t) - P_i(t)\mathcal{R}_i(t) \ \forall \ t \in [t_0, t_e].$$

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$$S_i^*(t) = S_i(t) - P_i(t)\mathcal{R}_i(t) \;\forall\; t \in [t_0, t_e].$$

- Therefore, given a probability density τ, for the arrival process $\mathcal{R}$, we can find the relationship between the risk premia (p) and the SETR (P),

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If an investor expects to receive a constant flow of premium $p_i^{t_0}$ from time t_0 through t_e, then the amount an investor expects to receive in premium is,

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We can see that the first integral on the right is the expected time of arrival of the risk and the second integral is 1

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where C is a normalisation factor

$$C = \frac{\int_{t_0}^{\infty} \tau(t_e) dt_e}{\int_{t_0+\Delta t}^{\infty} \tau(t_e) dt_e} = \frac{1}{\int_{t_0+\Delta t}^{\infty} \tau(t_e) dt_e}$$

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$$= C \int_{t_0}^{\infty} P_i(t_e)\tau(t_e)dt_e - C \int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e.$$

Modelling the SETR

From this, we can derive an expression for the expected price of the risk, if the transition event happens in the small interval $(t_0, t_0 + \Delta t)$. Accordingly,

$$p_i^{t_0+\Delta t}\left[\mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t\right] = C\int_{t_0}^{\infty} P_i(t_e)\tau(t_e)dt_e - C\int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e.$$

Making the substitution from equation (3), we obtain,

$$p_i^{t_0+\Delta t}\left[\mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t\right] = Cr_{t_0}\left[\mathbb{E}(t_e) - t_0\right] - C\int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e.$$

Modelling the SETR

$$\therefore \int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e = p_i^{t_0}\left[\mathbb{E}(t_e) - t_0\right] - \frac{p_i^{t_0+\Delta t}}{C}\left[\mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t\right]. \quad (4)$$

Modelling the SETR

$$\therefore \int_{t_0}^{t_0+\Delta t} P_i(t_e)\tau(t_e)dt_e = p_i^{t_0}\left[\mathbb{E}(t_e) - t_0\right] - \frac{p_i^{t_0+\Delta t}}{C}\left[\mathbb{E}(t_e|t_e > t_0 + \Delta t) - t_0 - \Delta t\right]. \quad (4)$$

Let $t_0 + \Delta t = t'$. Therefore, making this substitution in equation (4), we obtain,

$$\int_{t_0}^{t'} P_i(t_e)\tau(t_e)dt_e = p_{t_0}\left[\mathbb{E}(t_e) - t_0\right] - \frac{p_i^{t'}}{C}\left[C\int_{t'}^{\infty} t_e\tau(t_e)dt - t'\right].$$

Modelling the SETR

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$$\frac{\partial}{\partial t'}\left(\int_{t_0}^{t'} P_i(t_e)\tau(t_e)dt_e\right) = \frac{\partial}{\partial t'}\left[p_i^{t_0}\{\mathbb{E}(t_e) - t_0\} - p_i^{t'}\int_{t'}^{\infty} t_e\tau(t_e)dt - t'\int_{t'}^{\infty}\tau(t_e)dt_e\right]$$

$$\therefore P(t')\tau(t') = p_i^{t'}\left\{t'\tau(t') + \int_{t'}^{\infty}\tau(t_e)dt_e - t'\tau(t')\right\}.$$

Modelling the SETR

We can therefore write the value of the SETR explicitly in terms of the arrival process τ, as follows:

$$P(t') = \frac{p_i^{t'}}{\tau(t')} \int_{t'}^{\infty} \tau(x)dx. \tag{5}$$

Some Special Arrival Processes

We analyse the form of the SETR, just derived for some special arrival processes:

Some Special Arrival Processes

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- Exponential
- Uniform
- Gamma

Exponential Arrival Process

An exponential distribution with the parameter λ is given by

$$\tau(t) = \begin{cases} \lambda e^{-\lambda t} & t \geq t_0, \\ 0 & t < t_0. \end{cases}$$

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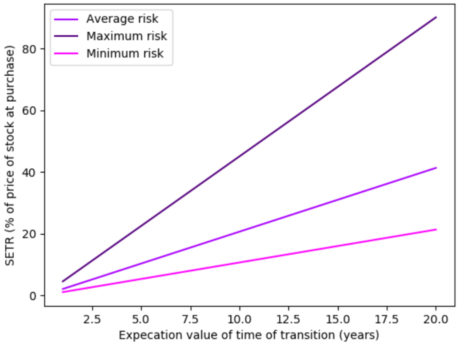
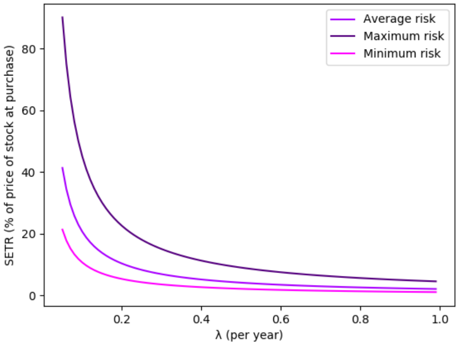
$$P_{\exp}(S_i, t') = \frac{p_i^{t'}}{\lambda}. \tag{6}$$

We find that the value of the SETR is independent of the time when the transition event occurs, and only depends on the value of the premium at the time of the transition. This happens because of the memoryless nature of the exponential distributions.

Exponential Arrival Process



(a) **Variation with** $\frac{1}{\lambda}$



(b) **Variation with** λ

Uniform Arrival Process

A uniform distribution with parameters θ_{min} and θ_{max} can be defined as

$$\tau(t) = \begin{cases} 0 & t < \theta_{\text{min}}, \\ (\theta_{\text{max}} - \theta_{\text{min}})^{-1} & \theta_{\text{min}} \leq t \leq \theta_{\text{max}}, \\ 0 & t > \theta_{\text{max}}. \end{cases}$$

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We find that unlike the exponential case, here, the SETR is a (linearly decreasing) function of the time of arrival.

Uniform Arrival Process

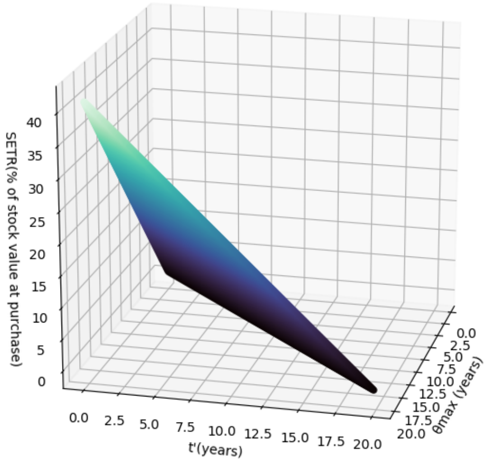

Figure: **Variation of SETR with $\theta_{\max}$ and time t'**

Gamma Arrival Process

We consider the Gamma arrival process with parameters a and b,

$$\tau(t) = \begin{cases} 0 & t < t_0, \\ \frac{b^a}{\Gamma(a)} t^{a-1} e^{-bt} & t \geq t_0. \end{cases}$$

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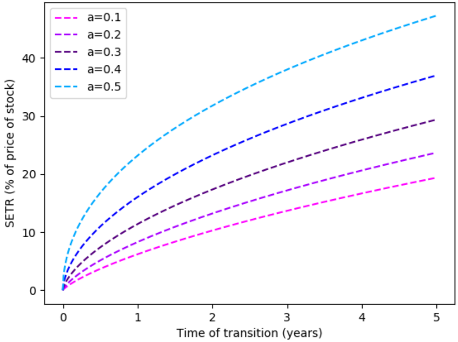
Therefore,

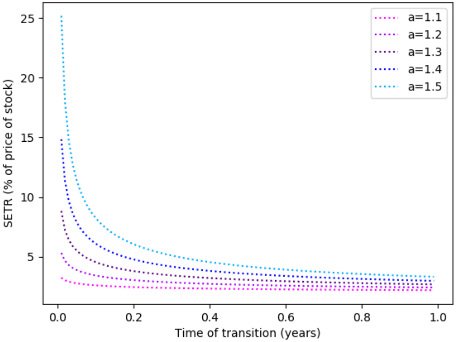
$$P(S_i, t') = \frac{p_i^{t'}}{b^a t'^{a-1} e^{-bt'}} \Gamma(a, bt'). \tag{8}$$

where $\Gamma(a, bt') = \int_{bt'}^{\infty} y^{a-1} e^{-y} dy$ is the upper incomplete gamma function.

Gamma Arrival Process



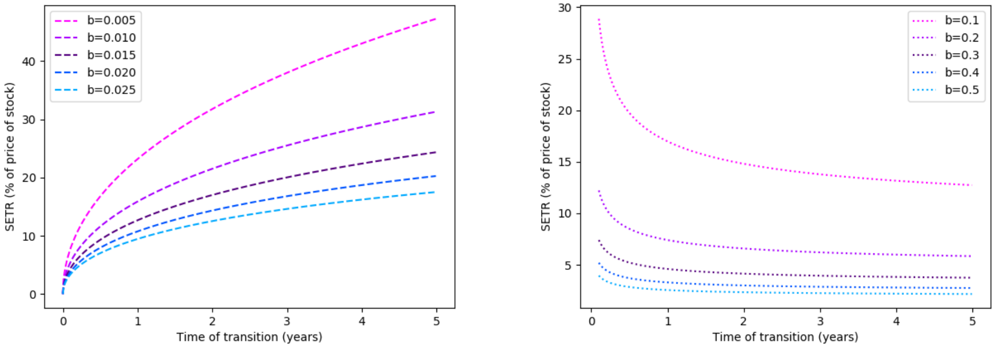
(a) b=0.005



(b) b=0.5

Figure: **Variation of SETR (% of price of stock at purchase) with time for $p_i^t = 1\%$**

Gamma Arrival Process



(a) a=0.5 (b) a=1.25

Figure: **Variation of SETR (% of price of stock at purchase) with time for** $p_i^t = 1\%$

Are the Premia Worth the Risk?

The condition for making profits from the premium-risk trade off is:

$$p_i(t_e - t_0) > \frac{p_i^{t'}}{\tau(t')} \int_{t'}^{\infty} \tau(x)dx.$$

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Therefore, in order to find our likelihood of making profits or losses, by taking long (or short) positions on the stocks of these companies, we need to find the area under the graph of the probability density function τ, for which this condition holds good. If the area is greater than 0.5, then the investor stands a better chance of making profits.

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Thus making the integration we get,

$$q_{\text{exp}} = \int_{\frac{1}{\lambda}}^{\infty} \lambda e^{-\lambda x} dx = \left[-e^{-\lambda x}\right]_{\frac{1}{\lambda}}^{\infty} = e^{-1} < 0.5.$$

Are the Premia Worth the Risk?

- **Uniform distribution**: For the uniform distribution, the SETR is not constant in time and is constantly varying, and, accordingly, the criterion for making profits is $p_i t_e > p_i(\theta_{\max} - t_e)$.

$$\therefore t_e = \frac{\theta_{\max}}{2}.$$

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Hence,

$$q_{\text{uni}} = \int_{\frac{\theta_{\max}}{2}}^{\theta_{\max}} \frac{1}{\theta_{\max}} dx = \frac{1}{2}.$$

Therefore, for a uniform arrival process, holding on to a polluting stock is probabilistically risk-neutral.

Are the Premia Worth the Risk?

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$$p_i t_e > \frac{p_i}{b^a t_e^{a-1} e^{-bt_e}} \Gamma(a, bt_e).$$

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Rearranging the terms, we finally obtain,

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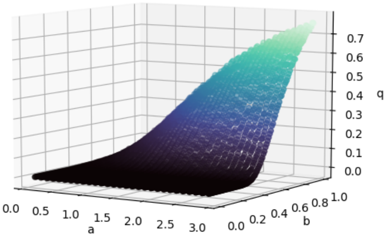
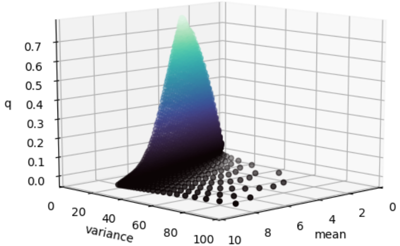
To determine the probability of positive returns, we generate large ensembles of t_e drawn from a $\Gamma(a, b)$ distribution for various combinations of a and b and estimate the probability in each case, by finding the proportion of cases in which the criterion holds.

Are the Premia Worth the Risk?



(a) Variation of q_γ with $\mu_\gamma, \sigma_\gamma^2$



(b) Variation of q_γ with a, b

Figure: **3D Scatter plot depicting the variation of q_γ**

Key Takeaways

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1. While stocks of highly polluting firms may look attractive in the short run, they pose significant risks on the long run.
2. This risk depends on the amount of emissions produced by the firms and the probability distribution of the arrival of the low-carbon transition event.
3. Despite the pricing of the risk being fair, the probability of making profits by investing in a polluting stock need not be even, for instance in the case of exponentially distributed arrival processes, the odds of making money is always lower than the odds of losing money.

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Thank You