Computers and Chemical Engineering 119 (2018) 215–227

Contents lists available at ScienceDirect

Computers and Chemical Engineering

journal homepage: www.elsevier.com/locate/compchemeng

Optimal operation of water distribution networks with intermediate storage facilities

Varghese Kurian[a], Saravanan Chinnusamy[a], Ashok Natarajan[b], Sridharakumar Narasimhan[a,*], Shankar Narasimhan[a]

[a] Department of Chemical Engineering, Indian Institute of Technology Madras, Chennai, India
[b] Tamil Nadu Water Investment Company, Chennai, India

ARTICLE INFO

Article history:
Received 27 January 2018
Revised 12 April 2018
Accepted 16 April 2018
Available online 8 May 2018

Keywords:
Water distribution networks
Energy minimization
Scheduling
Mixed integer non-linear programming

ABSTRACT

The nexus between water and energy reveals that transporting water for end use is a highly energy intensive operation. In this work we consider the optimal operation of a water distribution network consisting of pumps delivering water to different reservoirs, with each reservoir catering to a time varying demand. Pumps and ON/OFF valves are used as manipulated variables to minimize energy consumption while meeting the demand. Due to the nonlinear nature of the pump operating curve and the hydraulics, this results in a Mixed Integer Nonlinear Program (MINLP). We propose a three step decomposition approach to solve this problem efficiently. The applicability of this technique is demonstrated on a water network proposed for a municipality in India and the potential advantages are reported. We also compare the solution times required for the proposed technique and a standard solver and demonstrate the efficiency of the proposed approach.

1. Introduction

Water distribution networks (WDNs) are important infrastructural assets for human settlements. A typical network consists of storage reservoirs, pipes, pumps and valves, and is used to transport treated water from the source to consumers located in its region of service. It is estimated that water supply and treatment accounts for nearly 8% of global energy consumption (Garcia and You, 2016). Pumping cost accounts for a significant portion of the operational expenditure in WDNs (Bunn and Reynolds, 2009), and hence optimal operation of pumps holds the potential to significantly reduce the energy expenditure (Hong et al., 2017; Jowitt and Germanopoulos, 1992; Waterworth and Darbyshire, 2001). Determining this optimal schedule for operation of pumps is a difficult task for both managers and researchers alike. The variability in water consumption and the complexity of water distribution networks limits the use of heuristic solutions (López-ibáñez et al., 2008) and hence the recourse to formal mathematical optimization formulations. However, the nonlinear nature of the hydraulic equations and use of integer variables to model the state of the pumps and valves results in complex, high dimensional, nonlinear integer optimization problems which are known to be computationally demanding (Shi and You, 2016). Energy savings that can be achieved through pump scheduling and the computational complexity involved in obtaining this are well documented for domains outside water distribution as well (Cafaro et al., 2015; Magatao et al., 2004; Rejowski Jr. and Pinto, 2008).

One of the earliest reported approaches used a dynamic programming technique for solving this problem (DeMoyer and Horwitz, 1975). Tarquin and Dowdy (1989) reported a technique for selection of pumps considering operational efficiency as well. Subsequently, linearized network equations and constraints were used to determine the optimal schedule by Jowitt and Germanopoulos (1992). The applicability of the procedure depended on the validity of a few assumptions that decoupled network hydraulics from pump station controls and the technique was described to be useful in networks where the head lift given by pumps was large in comparison to the nodal head fluctuations caused by pump or valve switches elsewhere in the network. A dynamic programming approach which considers pump switches was introduced by Lansey and Awumah (1994), but the computational complexity associated with this procedure limited its applicability to only small and medium sized systems. Recently, a Lagrangian relaxation approach coupled with an improved limited discrepancy search was proposed by Ghaddar et al. (2015), and subsequently used in a combined design-operation problem put forth by Naoum-Sawaya et al. (2015). Researchers have also explored

* Corresponding author.
E-mail address: sridharkrn@iitm.ac.in (S. Narasimhan).

https://doi.org/10.1016/j.compchemeng.2018.04.017
0098-1354/

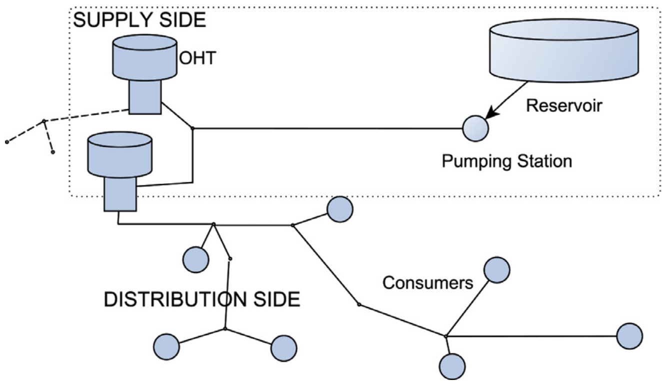


Fig. 1. Supply and distribution sides in an intermittent water system.

the possibility of implementing online controllers that make use of measurements available from the network (Sankar et al., 2015; Sopasakis et al., 2018). An aggregation of the various models that can be used under different assumptions has been carried out by Burgschweiger et al. (2009) and a survey of the mathematical programming techniques available for both network design and operation is presented by D'Ambrosio et al. (2015). Mala-Jetmarova et al. (2017) have comprehensively reviewed over 200 publications which are relevant to optimal operation of water distribution systems, particularly optimal pump operation, valve control and water quality and tabulated their findings.

In addition to the above mentioned mathematical programming techniques, evolutionary algorithms also have been applied towards solving the pump scheduling problem. A comparative study of the performance of various multi-objective evolutionary algorithms, namely SPEA, NSGA, NSGA 2, CNSGA, NPGA and MOGA was reported by Baran et al. (2005). The authors identified SPEA as the preferable technique for this application. A hybrid method which combined Genetic Algorithms (GAs) with Hill-climber strategies (Hookes and Jeeves and Fibonacci) was found to be superior to using GA alone in the works of Zyl et al. (2004). López-Ibáñez et al. (2005) introduced a new representation for a pump schedule using which it was possible to restrict the number of pump switches to a specified maximum while using GAs. More recently, neutral evolutionary search and ant colony optimization have also been used for solving the pump scheduling problem (López-ibáñez et al., 2008; Selek et al., 2012). Further, Bagirov et al. (2013) used a combination of grid search and Hookes and Jeeves method to solve the MINLP. A more prominent feature of the paper is that the authors used a continuous time formulation as against the discrete time formulation used by most of their peers.

In this work, we focus on a specific class of water distribution networks having the following characteristics. Water from the treatment plants is supplied to storage reservoirs (elevated or sumps) from which water is distributed (by gravity or pumping respectively) to end consumers. Here,"supply side" denotes pumps and pipelines delivering water from the treatment facility to the intermediate reservoirs (typically overhead tanks - OHTs) and the "distribution side" consists of pipes delivering water from these overhead tanks to the consumers as shown in Fig. 1. The inlet pipes to the OHTs are fitted with ON/OFF valves which are opened and closed in a prescribed schedule to manage the water supply. This design and operation is typically found in rural water supply schemes (Bhave and Gupta, 2006; Bonvin et al., 2017) and several urban water systems as well (Amrutur et al., 2016). The problem considered in the present work is to determine the optimal schedule for operation of pumps and valves for supplying water to the OHTs to satisfy the time varying demand requirements, while minimizing total energy consumption. There are very specific differences in design and operation of such water networks as opposed to well studied, urban water networks and we describe them in detail. Such systems are not well studied even though a significant number of rural water schemes follow these designs (Bhave and Gupta, 2006; Bonvin et al., 2017).

The first difference is that we consider fill and draw type reservoirs rather than floating reservoirs. In the fill and draw reservoir design, the inlet and outlets to the storage reservoirs are served by different pipes (World Bank, 2012). This allows hydraulic decoupling between the supply and distribution networks with the reservoirs serving as buffering tanks rather than to balance pressures. Though there has been a recent work on modeling of the filling process of such OHTs by Giustolisi et al. (2014), operation of such systems has received relatively less attention (Bonvin et al., 2017). Although not explicitly stated, the recent work on optimal operation of rural water supply schemes by Bonvin et al. (2017) implicitly assumes this design configuration. Here the authors have presented an efficient technique for scheduling pumps in drinking water networks operated by continuous control valves under few assumptions on pump characteristics. In another work, Amrutur et al. (2016) proposed a technique for identifying the possibility of continuous water supply in similar systems.

The second difference is in the operation of such systems. In some recent work on optimal operation of such networks (Amrutur et al., 2016; Bonvin et al., 2017), flow (continuous) control valves are used to regulate flows which have two disadvantages. Primarily, the installation and maintenance cost are much higher for the continuous control valves. Secondly, the output of the scheduling or optimization algorithm is usually a flow rate that must be maintained through the valve. Control valves are highly nonlinear with varying behaviour over the operating range. Hence, in order to achieve the desired flow, the control valve should be placed in a feedback loop with a flow measuring device to achieve the desired flow rate. Even though instrumentation costs are on the decline, cost and complexity of installing and maintaining these devices is high and hence the applicability in rural water networks in developing economies such as India seems limited.

An alternative strategy which we follow in this work is to use ON/OFF valves instead of continuous control valves. An extremely simple mode of operation is to open all valves in the inlet lines of the tanks simultaneously and close them manually as the tanks are filled (Bhave and Gupta, 2006). However, this is not the only possible solution. Given that the intermediate tanks in urban and rural schemes are sized to store as much as 30% and 50% of daily consumption, respectively (Venugopalan, 1999), manipulating the status of the valves and the times for which they are switched ON/OFF allows significant flexibility in operation. With availability of low cost, low power communication tools, an IoT based system to operate the valves in the desired manner can be implemented (Verma et al., 2015) at relatively lower costs as compared to using control valves.

The third difference is that most published work address the pump scheduling problem for WDNs with continuous supply as they are common in the developed economies. Though not prevalent in the developed countries, hundreds of millions of people around the globe continue to be served by intermittent water supply systems (Andey and Kelkar, 2009; Kumpel and Nelson, 2016; Vairavamoorthy et al., 2009). Shortage in supply and leaks in the system at times force the utility managers to intermittently operate networks which are inherently designed to provide continuous supply (De Marchis et al., 2010). In systems with continuous supply, the pipes are pressurized throughout the day and the consumers withdraw from the line according to their requirement. In intermittent supply systems, consumers are provided supply once

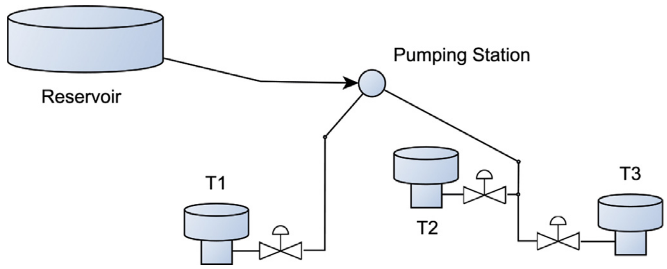


Fig. 2. A schematic of a supply network.

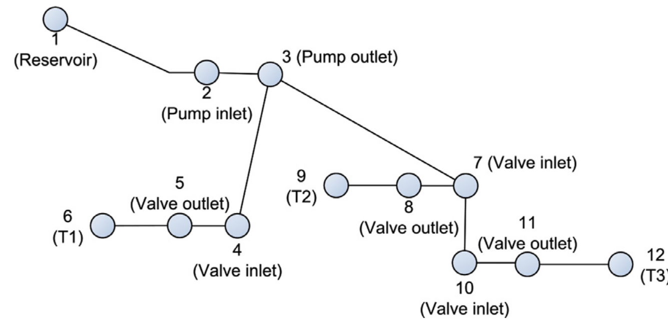


Fig. 3. Graphical representation of the network given in Fig. 2.

or twice in a day for a duration of few hours. The consumers draw enough water for the day during this periods and store it at their facility (Sankar et al., 2015; Wanjiru et al., 2016). Intermittent supply systems are mostly employed in areas where there is a need to limit the quantity of water supplied to consumers. The intermediate storage available in these systems affords significant flexibility in operating the network in comparison to the continuous supply systems where the nodal pressure constraints have to be met for all times of the day. The proposed technique is applicable to continuous and intermittent networks.

In the present work, we propose an efficient technique for arriving at the optimal policy for operating valves and pumps in such networks. Pumps and ON/OFF valves are used as manipulated variables to control the flow and pressure. The decision variables are the number of pumps to be turned on and the state of the valves in the network over a given horizon and the objective is to minimize energy consumption while meeting the time varying demand. Given the nonlinear nature of the system, this results in a Mixed Integer Nonlinear Program (MINLP). We propose to solve this by decomposing it into a series of sub-problems that can be solved efficiently. The first level of decomposition is to decouple the hydraulic simulations from the optimization. A set of hydraulic simulations are carried out and their solutions are passed on as parameters for the optimization problem. The remaining optimization problem is still a large dimensional integer problem and hence, we further decompose it into two sequential sub-problems. In the first sub-problem, we solve a relaxed Linear Program (LP) that ignores the time varying demand. In the subsequent sub-problem, an optimal schedule is determined after reinstating all constraints that were relaxed earlier. This results in an integer linear program (ILP) of a much smaller size (as compared to the original problem) and hence can be solved efficiently. Also the solution time does not scale up notably with the number of pipes and junctions in the system. For small or medium scale systems, the technique also provides the flexibility of completely avoiding hydraulic models by replacing it with few field measurements. The applicability of this technique is demonstrated on a municipal water supply network and the potential advantages are reported.

2. Problem description

The system in consideration here is the supply side of a water distribution network. A schematic of such a supply network delivering water to three OHTs from a single source is given in Fig. 2 and its equivalent graphical representation is given in Fig. 3. In the system, potable water is pumped from the source (treatment plant or water source) to OHTs, the withdrawals of which are assumed to follow a pattern that is known a priori. The source is assumed to be maintained at a constant head and the electricity tariffs are also assumed to be constant throughout the day. An ON/OFF valve is provided at the inlet of each tank using which the supply into the tank is controlled. The head against which the pump operates as well as the flow rates will depend on the set of OHTs that are receiving supply. In other words, the combination of valves that are opened together does affect the energy consumed by the pump. Therefore, the challenge here is to identify the optimal water filling schedule while meeting the consumer demands. The schedule can be equivalently described in terms of set of valves that are to be opened and pumps to be operated at different times of the day. Though the system in consideration here has only a single source and a single pump, the proposed technique can easily be extended to systems with multiple sources and pumps.

A structural feature of the supply systems considered here is the location of inlet to the OHTs. While continuous supply systems have a single line connected to the bottom of the reservoir serving as both the inlet and outlet from it, the systems under consideration (typically rural water supply schemes) have their inlet to the OHTs located at the top. Location of inlet at the top promotes mixing of the water during supply into the tank and thereby accumulation of sediments in the tank is prevented. This helps in maintaining the quality of water supplied downstream (Ainsworth, 2004). From a modelling perspective, this provides the advantage that the level of water in the tanks does not affect the head against which the pumps have to operate. The pumps always have to deliver to a height equal to the inlet of the OHTs, which is located at the top of the tanks, irrespective of the water level in it.

With these details, an optimization problem (P) is formulated for obtaining the schedule that delivers the water to the OHTs in an energy efficient manner. In the continuous time formulation discussed below, the horizon is broken down to I_{max} time slots each with their respective time span $\tau_1, \tau_2, \ldots, \tau_{I_{max}}$. The decision variables are the lengths of the time intervals, the valves and pumps to be operated in each of these intervals and correspondingly, the pressures and flows in the network in these intervals. Here we are searching for a cyclic schedule and hence starting level of water in the tanks are also decision variables. The parameters for the optimization problem (and their equivalent for the graph given in Fig. 3) are as follows:

n_v, the number of valves in the system 3
n_p, the number of pumps in the system 1
N, the set of junctions (nodes) in the system {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}
N_I, the set of intermediate nodes in the system {2, 3, 4, 5, 7, 8, 10, 11}
N_T, the set of tanks in the system {6, 9, 12}
s, the source of supply {1}
E, the set of arcs in the network. An arc may indicate a pipe, a valve or a pump {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (3, 7), (7, 8), (8, 9), (7, 10), (10, 11), (11, 12)}
E_{valve}, the set of arcs representing a valve {(4, 5), (7, 8), (10, 11)}
E_{pump}, the set of arcs representing a pump {(2, 3)}

E_{pipe}, the set of arcs representing a pipe {(1, 2), (3, 4), (5, 6), (3, 7), (8, 9), (7, 10), (11, 12)}
H_j, the capacity of jth tank
$z_{(l,\,m)}$, the length of pipe segment (l, m), $(l, m) \in E_{pipe}$
$d_{(l,\,m)}$, the diameter of pipe segment (l, m), $(l, m) \in E_{pipe}$
$\phi_{(l,\,m)}$, the Hazen Williams coefficient for the pipe segment (l, m), $(l, m) \in E_{pipe}$

The following are the variables used:

τ_i, the length of the ith time interval
$x_{i,\,(l,\,m)}$, the binary indicator variable for the valve positions/pump ON/OFF status in time slot i, $(l, m) \in E_{valve} \cup E_{pump}$
$Q_{i,\,(l,\,m)}$, the flow rate across arc (l, m) in time slot i, $(l, m) \in E$
$h_{i,\,m}$, the head at node m in time slot i
$\psi_{i,\,(l,\,m)}$, the head developed by the pump (l, m) in the ith time slot, $(l, m) \in E_{pump}$
$P_{i,\,(l,\,m)}$, the power consumed by the pump (l, m) in the ith time slot, $(l, m) \in E_{pump}$
C, the cost per unit energy consumed (assumed to be constant throughout the day)
$D_{i,\,j}$, the cumulative withdrawal from jth tank at the end of the ith time slot, $j \in N_T$, $1 \le i \le I_{max}$
$V_{i,\,j}$, volume of water in the jth tank at the end of the ith time slot

The complete formulation is as follows:

$$(P)\ \min_{x,\ \tau,\ V,\ Q,\ h} \sum_{i} \sum_{(l,m)\in E_{pump}} CP_{i,(l,m)}\tau_i \tag{1}$$

$$s.t.\ Q_{i,(l,j)}\tau_i - [D_{i,j} - D_{i-1,j}] = [V_{i,j} - V_{i-1,j}],\quad 1 \le i \le I_{max},\ j \in N_T,\ (l,j) \in E \tag{2}$$

$$\sum_{(l,m)\in E} Q_{i,(l,m)} = 0, \qquad 1 \le i \le I_{max},\ m \in N_I \tag{3}$$

$$Q_{i,(l,j)} \ge 0, \qquad 1 \le i \le I_{max},\ j \in N_T,\ (l,j) \in E \tag{4}$$

$$h_{i,j} = h_{i,jo}, \qquad 1 \le i \le I_{max},\ j \in N_T \tag{5}$$

$$h_{i,l} - h_{i,m} = \mathrm{sgn}(Q_{i,(l,m)})\frac{10.67\ |\ Q_{i,(l,m)}\ |^{1.85}\ z_{(l,m)}}{\phi_{(l,m)}^{1.85}\, d_{(l,m)}^{4.87}},\quad 1 \le i \le I_{max},\ (l,m) \in E_{pipe} \tag{6}$$

$$(h_{i,l} - h_{i,m})x_{i,(l,m)} = 0, \qquad 1 \le i \le I_{max},\ (l,m) \in E_{valve} \tag{7}$$

$$(h_{i,m} - h_{i,l})x_{i,(l,m)} = \psi_{i,(l,m)}, \qquad 1 \le i \le I_{max},\ (l,m) \in E_{pump} \tag{8}$$

$$0 \le V_{i,j} \le H_j, \qquad 0 \le i \le I_{max},\ j \in N_T \tag{9}$$

$$\sum_{i=1}^{I_{max}} Q_{i,(l,j)}\tau_i = D_{I_{max},j}, \qquad j \in N_T,\ (l,j) \in E \tag{10}$$

$$\sum_{i=1}^{I_{max}} \tau_i = 24 \tag{11}$$

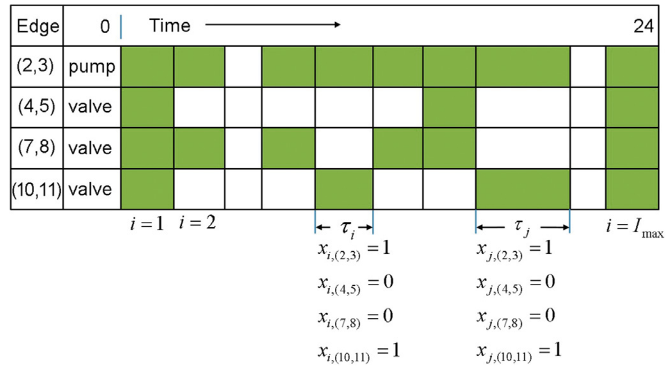


Fig. 4. A schedule for the network given in Fig. 2.

In addition, non-negativity constraints are imposed on all variables except Q. Here, Eqs. (2) and (3) are the flow balances for the tanks and other intermediate nodes of the network, respectively. Eq. (5) specifies the constant head at the OHTs and Eq. (6) models the pressure drop caused by friction across the different pipes in the network. Although Eq. (7) in its current form represents a valve with no pressure drop when fully open, it can be easily modified to include a resistance term if required. The head developed by the pump is given by Eq. (8) and the bounds on the volume of water in the OHTs are prescribed by Eq. (9). It has to be noted that, the problem is defined for a cyclic schedule and the initial level is also left as a free variable. Eqs. (10) and (11) ensure that the total daily demand of all OHTs are satisfied within the 24 h available. The right hand side (RHS) of Eq. (11) is set to 24 h under the assumption that the schedule is being prepared for a day. The appropriate horizon length may be substituted in case it is different. The solution to this problem gives the time span of different slots (τ_i) and the position of the valves and pumps in each of these time periods. A sample solution obtained on solving P for the network given in Fig. 3 is depicted as a Gantt chart in Fig. 4. Here each row corresponds to a pump or a valve in the network and columns correspond to time intervals, each of width τ_i. A coloured cell indicates that the corresponding device is ON.

The head developed across the pump and the efficiency at the point of operation is given by the pump characteristic curves. The characteristic curves for a pump described later in this paper is given in Fig. 11. The solid line represents the relation between the flow rate and the head developed across the pump and the dotted lines represent the efficiency. The head developed and the efficiency of operation (η) can be effectively represented as quadratic functions of the flow rate (Bonvin et al., 2017). Further, the power consumed by a pump is given by the following relation:

$$P_{(.)} = \frac{\psi_{(.)} \times Q_{(.)}}{\eta_{(.)}}$$

Apart from the equations describing the pump characteristics, the other sources of non-linearity in the MINLP described earlier are the hydraulic model equations relating the pressure drop across pipes to the flow rates through them (Eqs. (6) and (7)). These relations define the head loss to be proportional to $Q^{1.85}$ where Q is the flow rate though the pipe.

Adding to the difficulty imposed by the non-linearities, the problem has significant number of integer variables. The ON/OFF valves present in the pipes leading to the OHTs are represented by binary variables (x) which takes the value 1 when the valve is ON and 0 otherwise. Since there is one such variable for every valve, for every time slot, $i \le I_{max}$, the total number of binary variables that have to be dealt with is $(n_v + n_p)I_{max}$. These binary variables and the non-linear constraints together make this problem difficult to solve using a conventional MINLP solver.

3. Solution approach

As mentioned earlier, the general scheduling problem (P) is an MINLP that is difficult to solve. Here we propose a methodology for solving this problem by decomposing it into three tractable sub-problems. In the first step, the hydraulic simulation is decoupled from the optimization. This can be achieved by observing that for every choice of the binary variables (viz., state of valves and pumps), the flow rates and pressures in the network are uniquely determined and independent of τ_i. For the schedule shown in Fig. 4, during time intervals i and j the set of valves that are ON are the same, and correspondingly the flow rates and pressures are the same. The flow rates and pressures for each such possible combination of valve and pump states can be determined using a hydraulic model alone. We refer to each combination of valve and pump state as an overall system state, and the number of such system states is equal to $2^{(n_v+n_p)}$. It may be also noted that the power consumption for each system state can also be determined from the solution of the hydraulic simulation. Although, $2^{(n_v+n_p)}$ hydraulic simulations have to be performed, this enables the decoupling of the hydraulic simulation from the optimization, and hence eliminates the source of non-linearities from the subsequent problems.

Despite the above decoupling, the large number of binary variables still makes the problem difficult to solve and therefore the problem is further decomposed. In the second step, we solve a relaxed problem where the instantaneous demands at each OHT is ignored and only the cumulative demand for each OHT is considered. This is mathematically equivalent to ignoring the binary variables and hence the resulting problem is an LP. The solution to this LP identifies the system states that are energetically favourable and also determines the time interval of operation for each selected system state. It is implicitly assumed that the solution to the relaxed problem can be made feasible for the original problem by a proper choice of values for the binary variables. The physical significance of this relaxation and the rationale for proposing this assumption are explained in detail in the later sections.

The third step is the determination of a feasible schedule that satisfies the time varying demands subject to the capacity constraint of the OHTs. The system states and their operating time intervals identified in the second step are used in conjunction with other constraints to formulate an ILP to determine such a feasible schedule. If a feasible solution is obtained for the ILP, then this also implies that the optimal solution of the original problem is obtained, since the LP relaxation represents a lower bound on the optimal value of the original problem. In case a feasible solution for the ILP cannot be obtained, we seek a sub-optimal solution to the original problem by reformulating the LP in the second step and iterating.

Each of above three steps is explained in detail in the following section using the network in Fig. 2 as an example.

3.1. Characterization of system states

The state space refers to the set of values the process variables (e.g., flow, pressure, valve positions) can take. The inlets to the OHTs are located at the top and the flow rates into them are unaffected by the level of water. Hence, the only variables which decides the flow of water in the pipes are the status (ON/OFF) of pumps and valves in the network. The state of the system (except for the level in the OHTs) can therefore be completely characterized by the pump and valve positions in the network.

Table 1
An enumeration of all possible states of the network given in Fig. 2. The pump is ON for all states except state 1.

State	T1	T2	T3
1	0	0	0
2	**1**	0	0
3	0	**1**	0
4	0	0	**1**
5	**1**	**1**	0
6	0	**1**	**1**
7	**1**	0	**1**
8	**1**	**1**	**1**

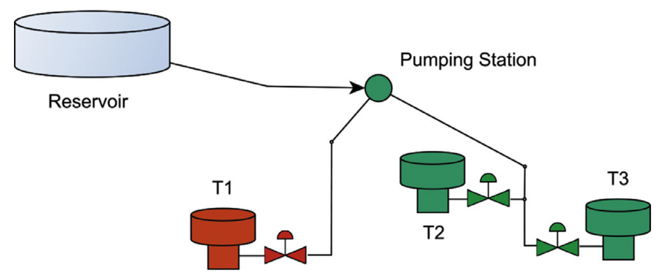


Fig. 5. A state of the network given in Fig. 2 with inlet valves of T2 and T3 ON and T1 OFF.

For the network shown in Fig. 2 with three tanks and one pump, the total number of possible states equals 2^4. However, the flow rates will be zero irrespective of the valve positions if the pump is turned off. Therefore, effectively only 2^3 states (two choices for each of the three valves, with the pump always ON) need to be considered. The complete set of possibilities is given in Table 1 and one of the states, namely state 6, is depicted in Fig. 5.

As the number of possible states for the system considered here are finite, and the flow rates in the pipes and the energy consumption are unique for a given state, the following notations are adopted for convenience:

$q_{j,\,k}$, the flow rate into tank j in state k and
c_k, the cost per unit time for operating the pump in system state k

From the solution of the hydraulic simulations, the values of $q_{j,\,k}$ and c_k can be determined. In our work, EPANET simulation software interfaced with MATLAB is used for the hydraulic simulations. Typically, rural water distribution networks have 10–12 OHTs and the computational requirements for performing these hydraulic simulations is still reasonable.

Although, we have used hydraulic simulations to determine the flow rates and energy consumption data for each system state, it is also possible to substitute these with measurements of these quantities obtained through field experiments. Hydraulic simulations require precise knowledge of the network structure and pipe characteristics (such as roughness coefficients) which may not be available, and the use of field measurements eliminates such modelling inaccuracies.

3.2. Optimal cost for operation

The overall optimization problem P can now be reformulated after removing all the non-linear constraints (6)–(8). Furthermore, the flow rates and power consumption can be treated as known parameters for each system state as determined from the hydraulic simulations. In order to reformulate the problem, we define binary variable $y_{i,\,k}$ which indicates whether the system state k is active or inactive in time interval i. The reformulated optimization problem P1 for minimizing energy consumption is as follows.

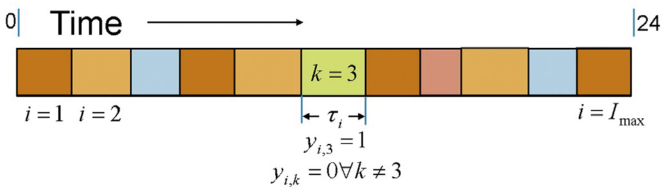


Fig. 6. A sample schedule after solving $P1$.

$$(P1) \quad \min_{y \in \{0,1\}, \tau, V} \quad \sum_i \sum_k c_k \; y_{i,k} \tau_i \tag{12}$$

$$\begin{aligned} s.t. \quad & \sum_p q_{j,k} \; y_{i,k} \tau_i - [D_{i,j} - D_{i-1,j}] \\ & = [V_{i,j} - V_{i-1,j}], \; 2 \le i \le I_{max}, \; j \in N_T \end{aligned} \tag{13}$$

$$0 \le V_{i,j} \le H_j, \qquad 0 \le i \le I_{max}, \; j \in N_T \tag{14}$$

$$\sum_{i=1}^{I_{max}} \sum_k q_{j,k} \; y_{i,k} \tau_i = D_{I_{max},j}, \qquad j \in N_T \tag{15}$$

$$\sum_{i=1}^{I_{max}} \tau_i = 24 \tag{16}$$

$$\sum_k y_{i,k} = 1, \qquad 1 \le i \le I_{max} \tag{17}$$

The optimal solution of the τ's and y's would give us a schedule similar to the one shown in Fig. 6. In this chart, each colour represent different states chosen by the optimization problem $P1$ and the width of each cell corresponds to the span (τ) of the particular time slot. As described earlier, each state corresponds to a configuration of the network that has to be active in the respective time slot. Further, a particular state may even be chosen for multiple time slots in the same day as shown in the figure. It has to be noted that the schedule shown here is only for illustration and indicative of the solution to $P1$ and not the actual solution of an optimization problem.

Even though problem $P1$ is a mixed integer linear program, the number of binary variables is $2^{(n_v+n_p)I_{max}}$. We formulate a simpler relaxed problem by ignoring constraints on the tank levels (Eq. (13)) which are required for meeting the time varying demands at each OHT. The rationality behind this relaxation is described is Section 4.1. The relaxed optimization problem after removing the time varying demand constraints is given below:

$$(P2) \quad \min_{y, \tau} \quad \sum_i \sum_k c_k \; y_{i,k} \tau_i \tag{18}$$

$$s.t. \quad \sum_k q_{j,k} \sum_{i=1}^{I_{max}} y_{i,k} \tau_i = D_{I_{max},j}, \qquad j \in N_T \tag{19}$$

$$\sum_{i=1}^{I_{max}} \tau_i = 24 \tag{20}$$

$$\sum_k y_{i,k} = 1, \qquad 1 \le i \le I_{max} \tag{21}$$

It has to be noted that $P2$ has no constraint connecting the ith time slot to the $(i-1)$th time slot. For any solution of $P2$, changing the order of time slots does not affect the feasibility or objective function. Therefore, for any optimal solution of $P2$, it is possible to reorder the time slots and obtain another optimal solution in which all time slots with the same system state appear consecutively. Consequently, the optimal solution to $P2$ determines how long each state is active in the day rather than the particular schedule. Hence, the following definition is adopted.

$$t_k = \sum_{i=1}^{I_{max}} y_{i,k} \tau_i$$

where t_k is the time span for which state k is active in the day. The optimization problem $P2$ can now be transformed into the following linear program (LP) with t_k as the decision variables

$$(P3) \quad \min_t \quad \sum_k c_k \; t_k \tag{22}$$

$$s.t. \quad \sum_k q_{j,k} \; t_k = D_{I_{max},j}, \qquad j \in N_T \tag{23}$$

$$\sum_k t_k = 24 \tag{24}$$

The solution of the optimization problem $P3$ includes the system states that are active during the day and the time interval for which each of these states is active, which results in least energy requirements. It can be observed that the number of constraints in the LP ($P3$) is $|N_T| + 1$. Thus, the optimal solution to the problem will have only $|N_T| + 1$ active system states in a day. This includes the trivial state in which the pump is OFF. The time span of these $|N_T| + 1$ states will add up to 24 h, and the solution is such that the total daily demand of every OHT is met and the energy consumed in the process is minimized.

It has to be noted that the solution of problem $P3$ satisfies the overall demand for each OHT. If the OHTs have sufficiently large capacities, then it is always possible to obtain a feasible schedule satisfying the instantaneous demand at each OHT using the solution identified by problem $P3$. In such a case, this feasible schedule also represents the optimal solution of the original problem P, since the optimal objective function value of $P3$ is a lower bound for the optimal solution of problem P. The following section describes the formulation of an ILP to find a feasible schedule for the given capacities of the OHTs and instantaneous demand requirements, using the solution obtained by $P3$

3.3. Search for a feasible schedule

In this step, the time of the day during which the different states are active is determined. Mathematically, this involves assigning appropriate values of y_{ik} such that Eqs. (13) and (14) are finally satisfied. Various approaches are possible to find a schedule that satisfies the constraints of $P1$ from the solution of $P3$. The approach followed here divides the time span t_k of each active state into smaller time slots of equal time length such that each slot has a minimum specified length, say at least half an hour, and then permute these smaller time slots to obtain a schedule which satisfies all constraints of $P1$. An example of division and rearrangement of five states is shown in Fig. 7. As the minimum specified length of a time slot increases, the difficulty in finding a feasible schedule also increases.

The following new notations are used in formulating the problem for finding a feasible schedule.
U, the set of states determined to be active from $P3$. Each element k in U is active for a span of time t_k.
m_k, the greatest integer such that $\frac{t_k}{m_k} \ge 0.5 \; \forall \; k \in U$

The number of event points in the day is given by $\sum_{k \in U} m_k$. As there is no more ambiguity in the number of time slots in the day, I_{max} is updated with this value.

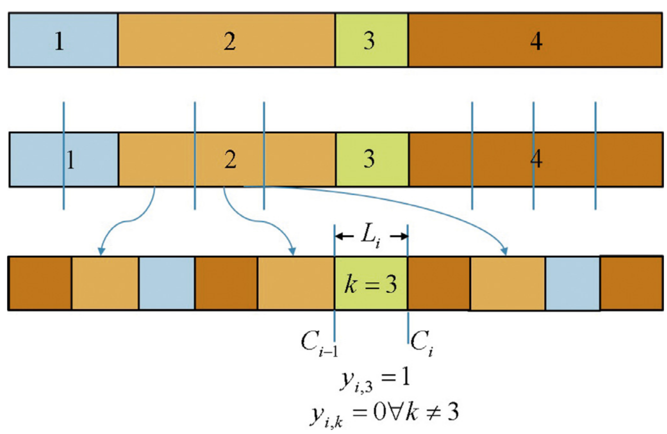


Fig. 7. An example showing the division and rearrangement of states.

The following new variables are also defined:

C_i, the completion time of time slot i, $1 \le i \le I_{max}$

L_i, the length of time slot i, $1 \le i \le I_{max}$

$f_{i,j,k}$, the quantity of water pumped into tank j in time slot i though pumping state k

$F_{i,j}$, the water pumped into tank j in time slot i

An optimization problem is formulated to determine a feasible schedule in which the time span of active states determined in $P3$ remain intact. As the program is a feasibility search, there is no objective function defined and the problem is only a set of constraints as described in $P4$. The constraints here are similar to those used by Cafaro and Cerda (2004) for the scheduling of a multi-product pipeline system.

$$(P4) \quad min \; 0$$

$$s.t. \; C_1 = L_1 \tag{25}$$

$$C_i = C_{i-1} + L_i, \qquad 2 \le i \le K \tag{26}$$

$$L_i = \sum_{k \in U} y_{i,k} \frac{t_k}{m_k}, \qquad 1 \le i \le I_{max} \tag{27}$$

$$f_{i,j,k} = q_{j,k} \; y_{i,k} \; L_i, \qquad 1 \le i \le I_{max}, \; j \in N_T, \; k \in U \tag{28}$$

$$F_{i,j} = \sum_{k \in U} f_{i,j,k}, \qquad 1 \le i \le I_{max}, \; j \in N_T \tag{29}$$

$$V_{i,j} - V_{i-1,j} = F_{i,j} - [D_{i,j} - D_{i-1,j}], \qquad 1 \le i \le I_{max}, j \in N_T \tag{30}$$

$$0 \le V_{i,j} \le H_j, \qquad 0 \le i \le I_{max}, j \in N_T \tag{31}$$

$$\sum_{k \in U} y_{i,k} = 1, \qquad 1 \le i \le I_{max} \tag{32}$$

$$\sum_{i=1}^{I_{max}} y_{i,k} = m_k, \qquad \forall \; k \in U \tag{33}$$

Here, Eqs. (26)–(29) are definitions of variables and Eq. (30) is the mass balance at the tank. Eq. (31) restricts the volume of water in each OHT to be within its capacity. The last two constraints ensures that only one state is active in a time slot, and every state is active for exactly the same time as determined by $P3$.

All the constraints except Eq. (28) are linear in the decision variables. Eq. (28) is bilinear since it involves the multiplication of a continuous variable with a boolean variable. However, it is possible to substitute this bilinear equality by three linear inequalities following the Glovers linearization scheme (Glover, 1975):

$$f_{(.)} \le L_{(.)} \; q_{(.)}$$
$$f_{(.)} \ge q_{(.)} \; (L_{(.)} - M(1 - y_{(.)}))$$
$$f_{(.)} \le M \; y_{(.)} \; q_{(.)}$$

where, M is a large number which can be set equal to the horizon length. If the binary variable $y_{(.)}$ is zero, the first two constraints become inactive and the last constraint ensures that $f_{(.)} = 0$. If $y_{(.)}$ is one, the first two constraints become active, and these inequalities reduces to the original equality constraint $f_{(.)} = L_{(.)} q_{()}$. The reason for using this reformulation is that it makes the formulation a mixed integer linear program (MILP) and can therefore be solved using a standard MILP solver.

A feasible solution to $P4$ is an optimal solution to the scheduling problem P with the optimal objective function value obtained in $P3$.

4. Addressing exceptions

The decomposition approach described in preceding section will provide an optimal solution in most cases. However, in some cases a feasible solution to problem $P4$ may not be obtained. This can arise due to the choice of the minimum time interval size used for formulating $P4$. We discuss a method to deal with such cases in the following subsections.

4.1. Feasibility of P4

The transformation of problem $P1$ to $P3$ is obtained by ignoring constraints on tank heights during the operation. In general, OHTs usually have sufficiently large capacities. The manual on water treatment and supply (Venugopalan, 1999) specifies that the capacity of service reservoir to be at least one third of the days demand for urban areas, and half of the daily demand in rural areas. However, in practice, the capacities are usually higher than these norms. Such high buffer capacities reduces the chances of OHTs overflowing or running dry in the day. Hence, the probability of obtaining the optimal solution to $P1$ by solving the relaxed problems $P3$ and $P4$ is high. However, in some cases, even though the original problem has an optimal solution, since $P3$ is a relaxation, we may get an infeasible solution for $P4$. If a feasible solution to $P4$ is not obtained, then we can reduce the minimum interval size from half an hour to a lower value. Reduction in interval size increases the number of intervals in the scheduling horizon and thereby improves the flexibility available for problem $P4$. At a very small interval length, $P4$ guarantees a feasible solution if the original problem P has a feasible solution. If we cannot reduce the minimum interval size below a limit (for practical reasons) and still do not have a feasible solution, then we will search for a suboptimal feasible solution. For this purpose, the Linear program $P3$ has to be revisited to find a new set of active states. This time, $P3$ is solved with a modification in the last constraint i.e, Eq. (24). The time allowed for completing the schedule is reduced to a value lower to than the time for which the pump was active in the previous solution of $P3$. Reducing the time allowable results in higher flow rates (pumping faster) and thereby finding a feasible schedule using $P4$ becomes easier. A flow chart describing the whole algorithm is given in Fig. 8.

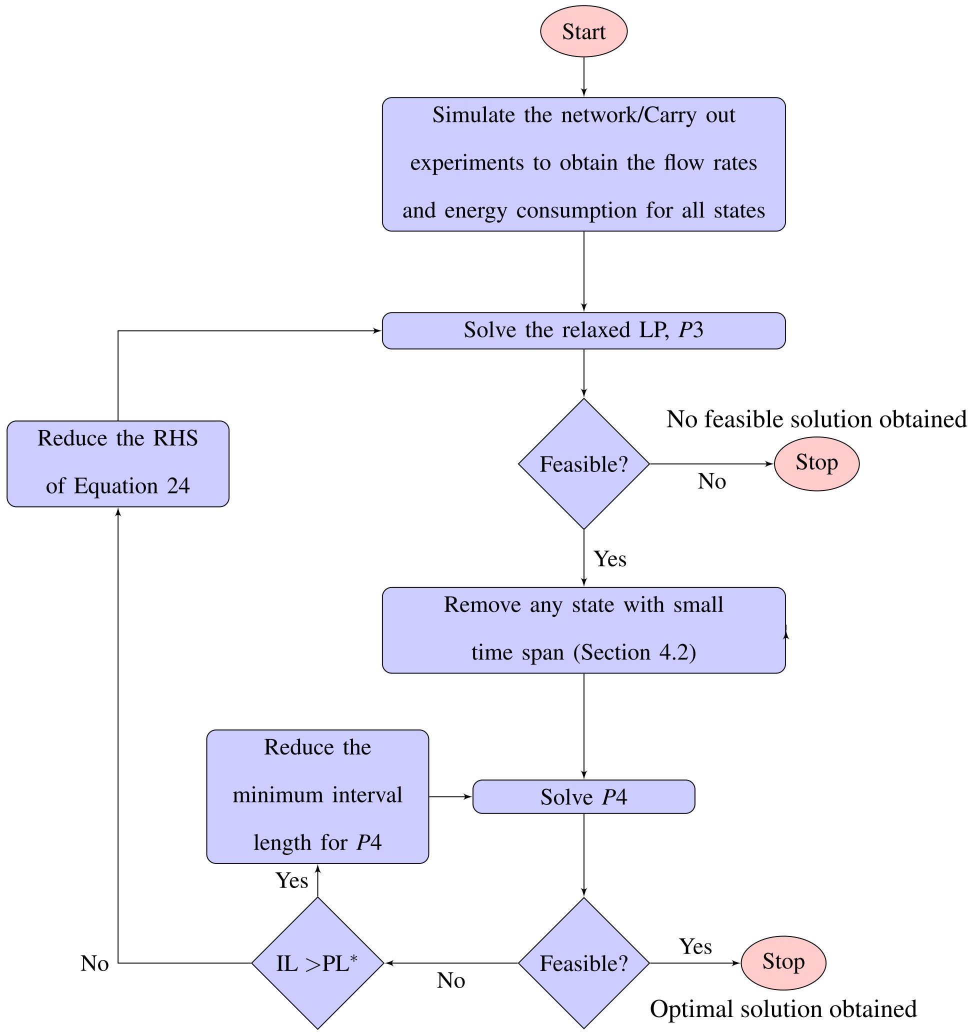


Fig. 8. Flow chart describing the complete solution procedure *IL: Minimum interval length, PL: Practical lower limit for interval length.

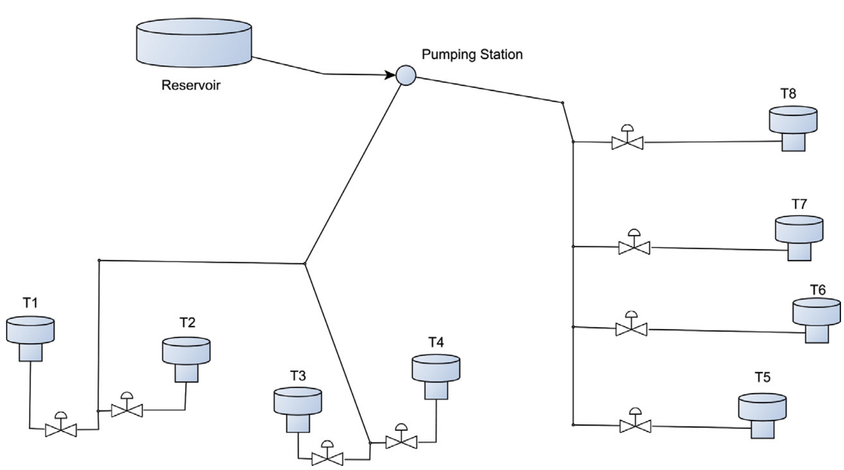


Fig. 9. Schematic of the supply network.

4.2. Active state with small time span in P3

The minimum length for time slots in $P4$ was taken as half an hour. This cannot be implemented if the total active time of any state t_k obtained from $P3$ itself is less than half an hour. In such situations, the state k has to be removed from the schedule after transferring the demand supplied by it into the remaining active states. That is, the particular state can be removed and other states can be given additional time such that the demand at all OHTs are satisfied. Performing this re-apportionment using system states which are already chosen as ‘efficient’ may not significantly increase the overall energy consumption.

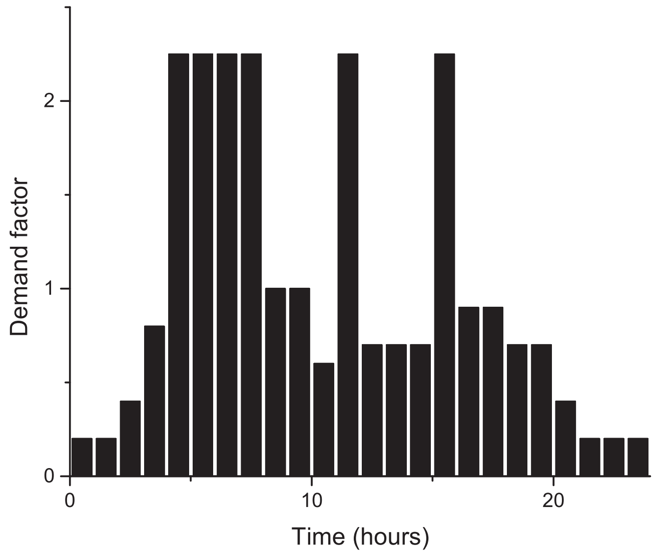


Fig. 10. Consumption pattern for the tanks.

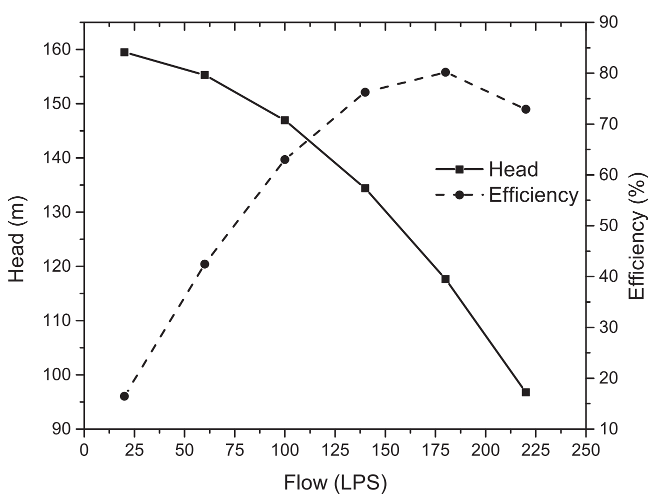


Fig. 11. Characteristic curves of the pump used.

4.3. *Initial tank level provided a priori*

In this technique, we assumed that the initial tank volumes are decision variables and the Problem *P*4 was provided with the freedom to choose their value (subject to the constraints). While implementing this schedule, the tank levels are brought to this value at the end of the first day and the schedule can be repeated the following day, if there are no changes to the demand patterns. However, there can be unexpected variations in consumption and

Table 2
Tank capacities and withdrawals.

Tank	Daily Demand (MLPD)	Capacity (ML)
T1	0.63	0.21
T2	0.77	0.257
T3	1	0.333
T4	1.3	0.433
T5	0.61	0.203
T6	3.08	1.027
T7	1.01	0.337
T8	0.72	0.24

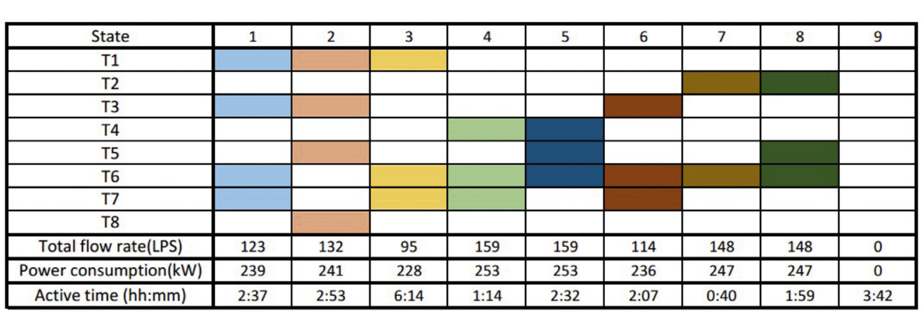
the daily schedule has to be adjusted depending on the initial water levels in OHTs. In such cases, the available initial tank levels have to be treated as a specified parameter rather than a decision variable for *P*4. It has to be noted that this can cause a slight reduction in the feasible region available for the problem.

5. Case study

A supply network that is being proposed for a city in southern India was selected for demonstrating the proposed approach. The network had a single pump (with standby) supplying water to eight overhead tanks from a single source. A schematic of the network is shown in Fig. 9. The daily withdrawal from the tanks and the capacities are given in Table 2. The withdrawal from the tanks followed the withdrawal pattern given in Fig. 10. The characteristic curves of the pump used in this study is given in Fig. 11.

In the first step, the network was simulated using EPANET for all the 2^8 configurations as described in Section 3.1. Each configuration was defined in MATLAB and EPANET was invoked multiple times from MATLAB to simulate the network. The computation time for carrying out the 256 network simulations was in the order of a few seconds. The flow rates into all tanks and the energy consumption rate for all the possible valve combinations were recorded. The pumping rate was the maximum for the state with all valves open (174.4 l/s) and this state also recorded the highest power consumption (257.7 kW). Notably, the energy consumed per unit of water supplied was the minimum for this state. The state with only T1 open recorded the lowest pumping rate of 17.6 l/s at a power consumption of 166.7 kW.

In the next step, problem *P*3 was formulated and solved for the network using MATLAB. *P*3 being a small LP, it took negligible time to complete. The selected states and their respective time spans are shown in Fig. 12, each colour indicating a unique state. As expected, the total number of active states was nine, including the trivial state where all valves were OFF. A colouration in any cell indicates that the valve to the tank is open in the corresponding state. The minimum energy required for supplying the water was identified to be 4853 kWh with pumping time slightly more than 20 h. The shortest state with only two tanks being supplied was active for 40 min while the longest state was active for



State	1	2	3	4	5	6	7	8	9
T1									
T2									
T3									
T4									
T5									
T6									
T7									
T8									
Total flow rate(LPS)	123	132	95	159	159	114	148	148	0
Power consumption(kW)	239	241	228	253	253	236	247	247	0
Active time (hh:mm)	2:37	2:53	6:14	1:14	2:32	2:07	0:40	1:59	3:42

Fig. 12. The active states and their time spans in the optimal schedule.

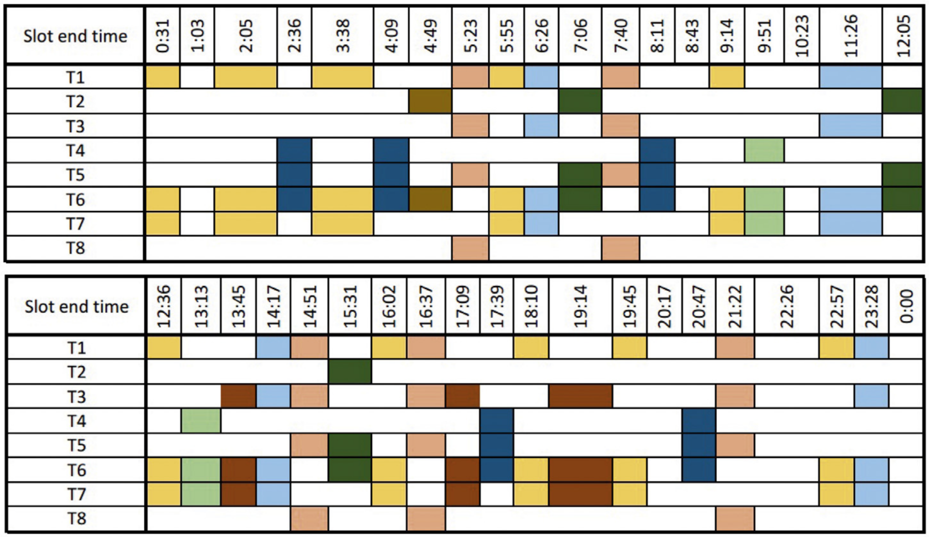


Fig. 13. The Gantt chart depicting the optimal schedule.

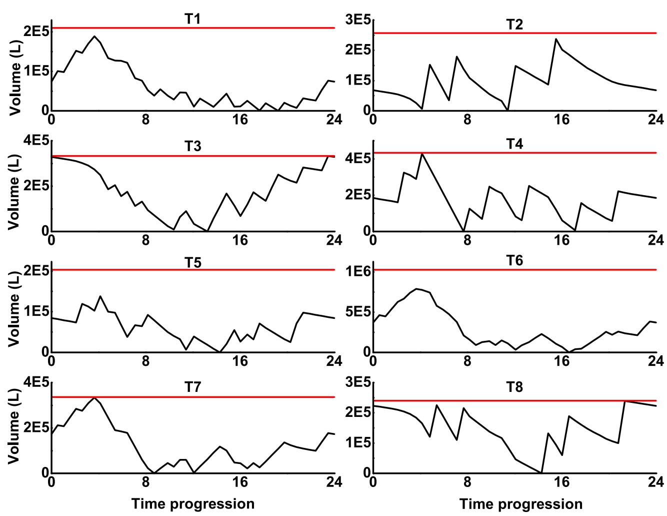


Fig. 14. The variation of levels in the OHTs following the optimal schedule. The straight lines indicate the maximum tank capacity.

more than six hours. It has to be noted that excepting state 7 and state 9 (when pump is OFF), at least 3 tanks are supplied with water. Also, the power consumed by the pump in these states were also similar. Following this, *P*4 was solved using CPLEX to obtain a feasible schedule. An Intel (R) Core (TM) 2 Duo CPU E7400 @ 2.80 GHz system solved the problem in 22 s. The Gantt chart depicting the optimal schedule is given in Fig. 13 and the level variations in the OHTs is given in Fig. 14. As pointed out earlier, this operation schedule required an energy consumption of 4853 kWh and pumping time of 20 hours 18 minutes in a day. The total number of valve operations required were 138. Analysing the schedule, an immediate question that may arise is why the second slot has pump OFF though most of the tanks have not filled to their respective maximum. The reason is that, though the OHTs are not filled at the end of the second slot, most of them almost reach their maximum sometime later. Therefore, if the pump is operated during the second slot as well, there is a chance that the OHTs may overflow at a later time instant.

For comparison, another schedule for the same system was developed heuristically. Two OHTs, T1 and the T6 were identified to be difficult to fill either due to high demand or elevated location. Therefore, the heuristic schedule was developed with only these two receiving the supply initially. Once the demand at any of the two were met, supply was provided to three of their neighbouring OHTs. Thereafter, the supply to a tank was closed as and when the total demand requirement of the tank was met. The active states identified in this way and their respective time spans are given in Fig. 15. As one of the states active in this policy of supply had only a time span of about 3 min, the state was removed and the supply was taken over by other active states as described in Section 4.2. A schedule was then obtained by solving problem *P*4 for these states as given in Fig. 16. This policy of operation had an energy expense of 5351 kWh and a pump operating time of 22 h 50 min per day.

The costs incurred for supplying water to only one tank at a time was also calculated. The energy required for this, 8566 kWh, was almost double as that required by the optimal schedule. Moreover, it was impossible to implement this schedule as it required a pumping time of over 40 h a day.

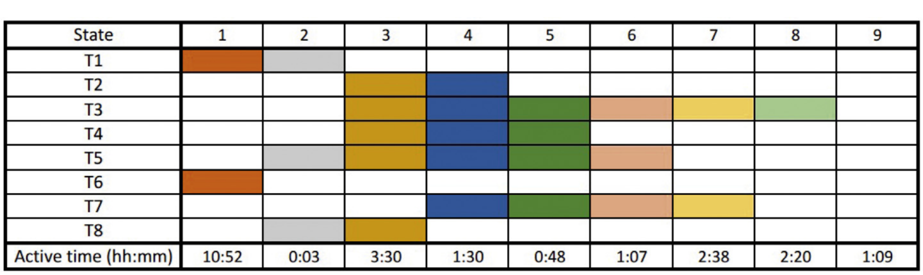


Fig. 15. The states identified for the heuristic schedule.

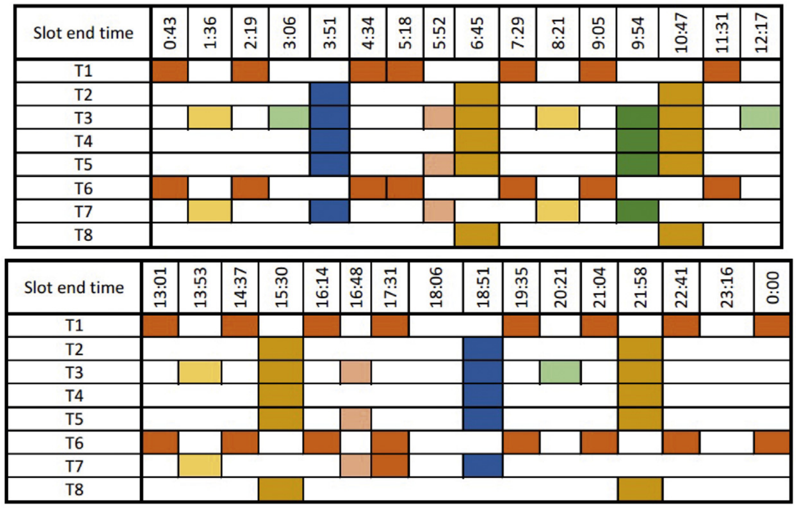


Fig. 16. The Gantt chart depicting the heuristic schedule.

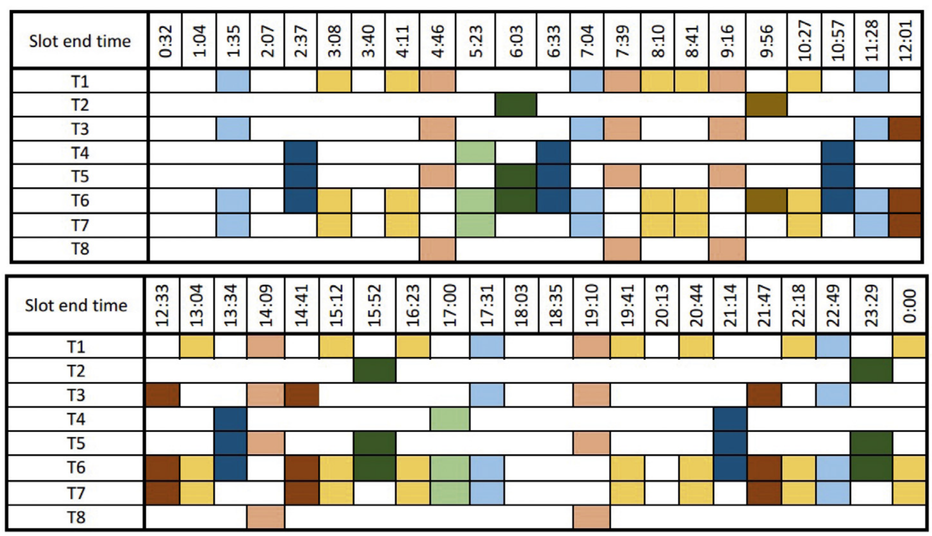


Fig. 17. The gantt chart depicting the optimal schedule with reduced tank capacity.

The time taken to solve the problem P without decomposition using a standard solver was also tested. For this purpose, rather than solving the complete non-linear problem (P) on an MINLP solver, the simpler version $P1$ was solved using an MILP solver. To remove the non-linearities completely, the day was discretized into intervals of equal length. Two cases were solved using CPLEX on the NEOS server (Czyzyk et al., 1998; Dolan, 2001; Gropp and Moré, 1997), one with interval length half hour and the other with interval length one hour. The first case provided a solution of 4854.7 kWh with an MIP gap of 0.85% in 1375 s. The second case with larger interval remained at an MIP gap of 1.88% even after 4500 s. On choosing CBC as the solver, in both the cases the program terminated with no results after the maximum running time of 8 h. This clearly demonstrates the efficiency of our proposed decomposition approach and its ability to find the optimal solution.

In the optimal schedule obtained (Figs. 13, 14), the water level in few OHTs were reaching their respective maximum and minimum permissible values at some instances. Such a schedule will not be able to accommodate any unexpected variations in demand. In order to overcome this, the constraint for $P4$ given by Eq. (31) was further tightened. The minimum permissible volume

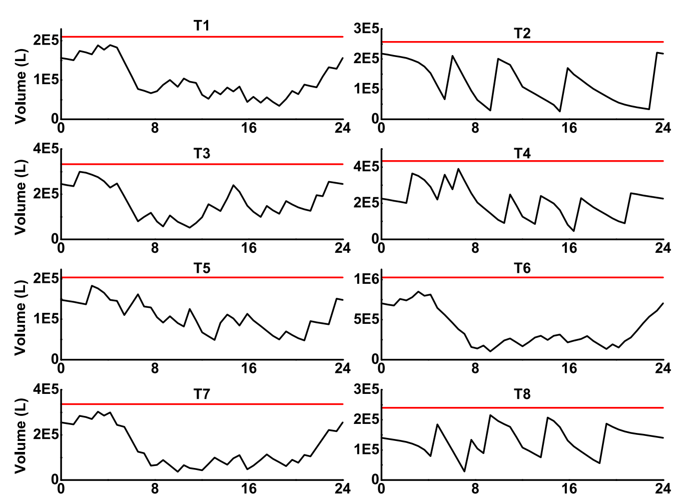


Fig. 18. The variation of levels in the OHTs with reduced capacity. The straight lines indicate the maximum tank capacity.

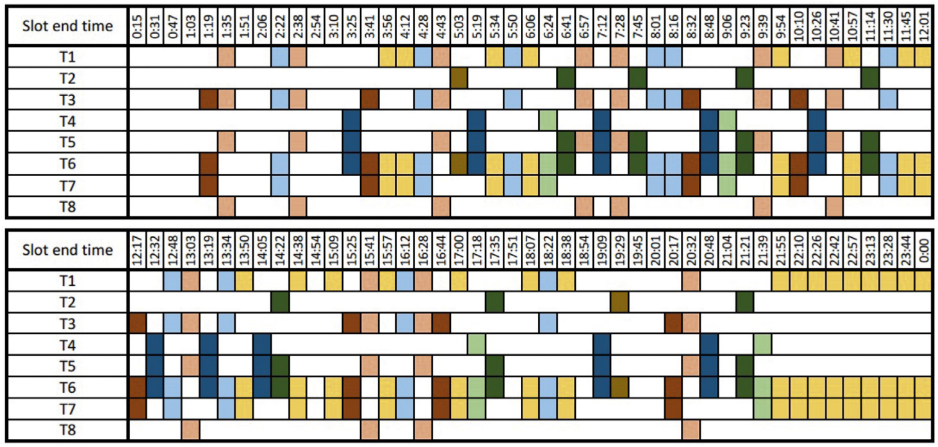


Fig. 19. The gantt chart depicting schedule with minimum interval length 15 min.

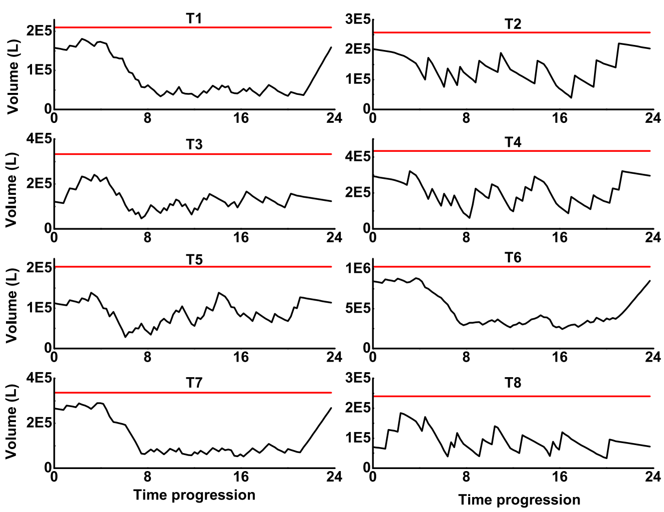


Fig. 20. The variation of levels in the OHTs with minimum interval length 15 min.

was increased to 10% of the tank capacity and the maximum permissible volume was reduced to 90% of the tank capacity. With this updated constraint, *P*4 was solved once again (in 132 s) and the resultant schedule is given in Fig. 17. Also, the variations in tank levels following this schedule is given in Fig. 18. It has to be noted that such a change in *P*4 to tighten the allowable limits of tank levels do not affect the objective function of the overall formulation.

The problem was further tightened by reducing the tank capacities by an additional 8%. This resulted in problem *P*4 becoming infeasible. This is addressed using the technique described in Section 4.1 As described earlier, to overcome this, the minimum interval length of *P*4 was brought down to 15 min from the earlier 30 min. The updated problem *P*4 was solved with ease and the resulting schedule is given in Fig. 19 and the level variation in the OHTs are given in Fig. 20.

In intermittent water systems, water is withdrawn from the OHT to smaller tanks rather than directly to consumption points (Wanjiru et al., 2016). Therefore, withdrawals normally stay close to a mean value throughout the day. Finding solutions of *P*4 for networks with constant withdrawal patterns is much easier than with withdrawal patterns of high variability. In the case study, a pattern provided in the Handbook on works audit (Office of the Principal Accountant General) for household consumption was used to model the withdrawal from the OHTs and this had high variance about the average daily demand. Therefore, the case taken for study here was undoubtedly one from the category of 'difficult' networks as far as the solution technique is concerned.

6. Conclusion

This paper discussed a novel technique for optimal operation of the supply side of a certain class of water networks, which are typical of low and middle income countries. The approach involved decomposing the MINLP into three smaller (and simpler) sub-problems which were solved in a sequence. The decomposition was made possible by exploiting few unique structural and operational features of the supply systems with intermediate storage facilities. The applicability of the technique was demonstrated on the model of a proposed network. The optimal schedule consumed 9.3% less energy in comparison to another schedule developed using heuristics. The daily pump operating time was also less for the optimal schedule by about 2.5 h in comparison with the heuristic schedule. Possible extensions of the technique may include incorporating time varying electricity tariffs for the system.

Acknowledgements

This work was partially supported by the Department of Science and Technology, Govt. of India under the Water Technology Initiative (Proj. No. DST/TM/WTI/2K13/144) and the IIT Madras Interdisciplinary laboratory for data sciences (CSE/14-15/831/RFTP/BRAV).

References

Ainsworth, R. (Ed.), 2004. Safe Piped Water. IWA Publishing (on behalf of the World Health Organization).
Amrutur, B., Kumar, M.M., Kumar, K.S., Patel, L., Sundaresan, R., Vaidhiyan, N.K., 2016. Wateropt: a method for checking near-feasibility of continuous water supply. In: Proceedings of the 2nd International Workshop on Cyber-Physical Systems for Smart Water Networks. IEEE, pp. 7–12.
Andey, S.P., Kelkar, P.S., 2009. Influence of intermittent and continuous modes of water supply on domestic water consumption. Water Resour. Manage. 23 (12), 2555–2566. doi:10.1007/s11269-008-9396-8.
Bagirov, A.M., Barton, A.F., Mala-jetmarova, H., Nuaimat, A.A., Ahmed, S.T., Sultanovaa, N., Yearwood, J., 2013. An algorithm for minimization of pumping costs in water distribution systems using a novel approach to pump scheduling. Math. Comput. Model. 57 (3–4), 873–886. doi:10.1016/j.mcm.2012.09.015.
Baran, B., Lucken, C.V., Sotelo, A., 2005. Multi-objective pump scheduling optimisation using evolutionary strategies. Adv. Eng. Software 36, 39–47. doi:10.1016/j.advengsoft.2004.03.012.
Bhave, P., Gupta, R., 2006. Analysis of Water Distribution Networks. Narosa Publishing House Pvt. Ltd.
Bonvin, G., Demassey, S., Le Pape, C., Maïzi, N., Mazauric, V., Samperio, A., 2017. A convex mathematical program for pump scheduling in a class of branched water networks. Appl. Energy 185, 1702–1711.

Bunn, S.M., Reynolds, L., 2009. The energy-efficiency benefits of pump-scheduling optimization for potable water supplies. IBM J. Res. Dev. 53 (3), 5:1–5:13. doi:10.1147/JRD.2009.5429018.

Burgschweiger, J., Gnadig, B., Steinbach, M.C., 2009. Optimization models for operative planning in drinking water networks. Optim. Eng. 10 (1), 43–73. doi:10.1007/s11081-008-9040-8.

Cafaro, D.C., Cerda, J., 2004. Optimal scheduling of multiproduct pipeline systems using a non-discrete milp formulation. Comput. Chem. Eng. 28 (10), 2053–2068. Special Issue for Professor Arthur W. Westerberg

Cafaro, V.G., Cafaro, D.C., Mendez, C.A., Cerda, J., 2015. Minlp model for the detailed scheduling of refined products pipelines with flow rate dependent pumping costs. Comput. Chem. Eng. 72, 210–221. A Tribute to Ignacio E. Grossmann. https://doi.org/10.1016/j.compchemeng.2014.05.012

CPLEX, IBM ILOG CPLEX optimization studio. https://www.ibm.com/us-en/marketplace/ibm-ilog-cplex Accessed: 2017-08-01.

Czyzyk, J., Mesnier, M.P., Moré, J.J., 1998. The NEOS server. IEEE Journal on Computational Science and Engineering 5 (3), 68–75.

De Marchis, M., Fontanazza, C.M., Freni, G., La Loggia, G., Napoli, E., Notaro, V., 2010. A model of the filling process of an intermittent distribution network. Urban Water J. 7 (6), 321–333. doi:10.1080/1573062X.2010.519776.

DeMoyer, R., Horwitz, Lawrence, B.j. a., 1975. A system approach to water distribution modeling and control. Lexington, Mass. : Lexington Books. First edition

Dolan, E.D., 2001. The NEOS Server 4.0 Administrative Guide. Technical Memorandum. Mathematics and Computer Science Division, Argonne National Laboratory.

D'Ambrosio, C., Lodi, A., Wiese, S., Bragalli, C., 2015. Mathematical programming techniques in water network optimization. Eur. J. Oper. Res. 243 (3), 774–788.

Garcia, D., You, F., 2016. The water-energy-food nexus and process systems engineering: a new focus. Comput. Chem. Eng. 91, 49–67.

Ghaddar, B., Naoum-sawaya, J., Kishimoto, A., Taheri, N., Eck, B., 2015. A Lagrangian decomposition approach for the pump scheduling problem in water networks. Eur. J. Oper. Res. 241 (2), 490–501. doi:10.1016/j.ejor.2014.08.033.

Giustolisi, O., Berardi, L., Laucelli, D., 2014. Modeling local water storages delivering customer demands in WDN models. J. Hydraul. Eng. 140 (1), 89–104.

Glover, F., 1975. Improved linear integer programming formulations of nonlinear integer problems. Manage. Sci. 22 (4), 455–460.

Gropp, W., Moré, J.J., 1997. Optimization environments and the neos server. In: Buhman, M.D., Iserles, A. (Eds.), Approximation Theory and Optimization. Cambridge University Press, pp. 167–182.

Hong, S.-P., Kim, T., Lee, S., 2017. A precision pump schedule optimization for the water supply networks with small buffers. Omega.

Jowitt, P.W., Germanopoulos, G., 1992. Optimal pump scheduling in water-supply networks. J. Water Resour. Plann. Manage. 118 (4), 406–422.

Kumpel, E., Nelson, K.L., 2016. Intermittent water supply: prevalence, practice, and microbial water quality. Environ. Sci. Technol. 50 (2), 542–553. doi:10.1021/acs.est.5b03973.

Lansey, K.E., Awumah, K., 1994. Optimal pump operations considering pump switches. J. Water Resour. Plann. Manage. 120 (1), 17–35.

López-Ibáñez, M., Prasad, T.D., Paechter, B., 2005. Optimal pump scheduling: representation and multiple objectives. In: Conference on Computing and Control for the Water Industry.

López-ibáñez, M., Prasad, T.D., Paechter, B., 2008. Ant colony optimization for optimal control of pumps in water distribution networks. J. Water Resour. Plann. Manage. 134 (4), 337–346.

Magatao, L., Arruda, L., Jr, F.N., 2004. A mixed integer programming approach for scheduling commodities in a pipeline. Comput. Chem. Eng. 28 (1-2), 171–185. Escape 12. https://doi.org/10.1016/S0098-1354(03)00165-0

Mala-Jetmarova, H., Sultanova, N., Savic, D., 2017. Lost in optimisation of water distribution systems? A literature review of system operation. Environmental Modelling & Software 93, 209–254.

Naoum-Sawaya, J., Ghaddar, B., Arandia, E., Eck, B., 2015. Simulation-optimization approaches for water pump scheduling and pipe replacement problems. Eur. J. Oper. Res. 246 (1), 293–306.

Office of the Principal Accountant General (Civil Audit), C.,. Handbook on Water Supply. Tamil Nadu Water Supply Drainage board.

Rejowski Jr., R., Pinto, J., 2008. A novel continuous time representation for the scheduling of pipeline systems with pumping yield rate constraints. Comput. Chem. Eng. 32 (4-5), 1042–1066. Festschrift devoted to Rex Reklaitis on his 65th Birthday. https://doi.org/10.1016/j.compchemeng.2007.06.021

Sankar, G.S., Kumar, S.M., Narasimhan, S., Narasimhan, S., Bhallamudi, S.M., 2015. Optimal control of water distribution networks with storage facilities. J. Process Control 32, 127–137.

Selek, I., Bene, J.G., Hos, C., 2012. Optimal (short-term) pump schedule detection for water distribution systems by neutral evolutionary search. Appl. Soft Comput. 12, 2336–2351. doi:10.1016/j.asoc.2012.03.045.

Shi, H., You, F., 2016. Energy optimization of water supply system scheduling: novel minlp model and efficient global optimization algorithm. AIChE J. 62 (12), 4277–4296.

Sopasakis, P., Sampathirao, A.K., Bemporad, A., Patrinos, P., 2018. Uncertainty-aware demand management of water distribution networks in deregulated energy markets. Environmental Modelling & Software 101, 10–22.

Tarquin, A.J., Dowdy, J., 1989. Optimal pump operation in water distribution. J. Hydraul. Eng. 115 (2), 158–168.

Vairavamoorthy, K., Gorantiwar, S.D., Mohan, S., 2009. Intermittent water supply under water scarcity situations. Water Int. 32 (1), 121–132. doi:10.1080/02508060708691969.

Venugopalan, V., 1999. Manual on Water Supply and Treatment. Central Public Health and Environmental Engineering Organization, Government of India.

Verma, P., Kumar, A., Rathod, N., Jain, P., Mallikarjun, S., Subramanian, R., Amrutur, B., Kumar, M.S.M., Sundaresan, R., 2015. Towards an iot based water management system for a campus. In: Proceedings of the IEEE 1st International Smart Cities Conference (ISC2), Mexico doi:10.1109/ISC2.2015.7366152.

Wanjiru, E.M., Zhang, L., Xia, X., 2016. Model predictive control strategy of energy-water management in urban households. Appl. Energy 179, 821–831.

Waterworth, G., Darbyshire, K., 2001. Comparison of methods of pump scheduling in water supply systems. European Simulation Multiconference.

World Bank, 2012. Rural Water Supply : Design Manual. Technical Report.

Zyl, J.E.V., Savic, D.A., Walters, G.A., 2004. Operational optimization of water distribution systems using a hybrid genetic algorithm. J. Water Resour. Plann. Manage. 130 (2), 160–170.