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BOOK OF ABSTRACTS

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Multiplicity results for a subcritical Hamiltonian system with concave-convex nonlinearities

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We study the *Hamiltonian elliptic system*

$$\begin{cases} -\Delta u = \lambda |v|^{r-1}v + |v|^{p-1}v & \text{in } \Omega, \\ -\Delta v = \mu |u|^{s-1}u + |u|^{q-1}u & \text{in } \Omega, \\ \quad u > 0, \ v > 0 & \text{in } \Omega, \\ \quad u = v = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a smooth bounded domain, λ and μ are nonnegative parameters and $r, s, p, q > 0$ with $r < p$, s, q and p, q in the subcritical regime with respect to the critical hyperbola.

Our study includes the case in which the nonlinearities in (1) are concave near the origin and convex near infinity. Motivated mainly by some of the results from [2], we focus on the region of non-negative *pairs of parameters* (μ, λ) that guarantee existence and multiplicity of solutions of (1).

Along the way, we discuss briefly a Brezis-Nirenberg type result for a fourth order boundary value problem in the sub-critical regime. This result is motivated by the seminal and celebrated result due to Brezis and Nirenberg in [3].

This is a work in collaboration with Bernhard Ruf (Universita Statale di Milano) and Carlos Vélez (Universidad Nacional de Colombia Sede Medellín) ([1]).

References

[1] O. Agudelo, B. Ruf, C. Velez. Multiplicity results for a subcritical Hamiltonian system with concave-convex nonlinearities. https://arxiv.org/abs/2412.10812

[2] A. Ambrosetti, H. Brezis, G. Cerami. Combined effects of concave and convex nonlinearities in some elliptic problems. *Journal of Functional Analysis.* 1994 Jun 1;**122**(2):519-43.

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Fractional Schrödinger equations with mixed nonlinearities

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We study the fractional Schrödinger equations with a vanishing parameter:

$$(\mathcal{P}_\lambda) \qquad \begin{cases} (-\Delta)^s u + u = |u|^{p-2}u + \lambda |u|^{q-2}u \text{ in } \mathbb{R}^N \\ u \in H^s(\mathbb{R}^N), \end{cases}$$

where $s \in (0,1)$, $N > 2s$, $2 < q < p \leq 2^*_s = \frac{2N}{N-2s}$ are fixed parameters and $\lambda > 0$ is a vanishing parameter. For λ small, we investigate the asymptotic behaviour of positive ground state solutions when p is subcritical, or critical Sobolev exponent 2^*_s. For $p < 2^*_s$, the ground state solution asymptotically coincides with unique positive ground state solution of $(-\Delta)^s u + u = u^p$, whereas for $p = 2^*_s$ the asymptotic behaviour of the solutions, after a rescaling, is given by the unique positive solution of the nonlocal critical Emden-Fowler type equation. Additionally, for $\lambda > 0$ small, we show the uniqueness and nondegeneracy of the positive ground state solution using these asymptotic profiles of solutions.

This is a joint work with Paramananda Das and Debdip Ganguly.

Payne nodal set conjecture for the fractional p-Laplacian in Steiner symmetric domains

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Let u be either a second eigenfunction of the fractional p-Laplacian or a least energy nodal solution of the equation $(-\Delta)_p^s u = f(u)$ with superhomogeneous and subcritical nonlinearity f, in a bounded open set Ω and under the nonlocal zero Dirichlet conditions. Assuming that Ω is Steiner symmetric, we show that the supports of positive and negative parts of u intersect $\partial\Omega$, and, consequently, the nodal set of u has the same property as long as Ω is connected.

The proof involves the analysis of certain polarization inequalities related to positive and negative parts of u, and alternative characterizations of second eigenfunctions and least energy nodal solutions.

The talk is based on the work [1].

References

[1] Bobkov, V., Kolonitskii, S. Payne nodal set conjecture for the fractional p-Laplacian in Steiner symmetric domains. Zapiski POMI, 536 (2024) 96-125. http://ftp.pdmi.ras.ru/pub/publicat/znsl/v536/p126.pdf

Regularity and explicit L^∞ estimates for a class of elliptic systems

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We consider a system of two linear decoupled Elliptic equations that are coupled on the boundary with a nonlinear boundary condition. We use De Giorgi-Nash-Moser iteration scheme to establish that weak solutions of the system with critical growth on the boundary are in $L^\infty(\Omega)$. Moreover, we provide an explicit $L^\infty(\Omega)$ estimate of weak solutions with subcritical growth on the boundary, in terms of powers of $H^1(\Omega)$-norms, by combining the elliptic regularity of weak solutions with Gagliardo–Nirenberg interpolation inequality.

The talk is based on the joint work with Nsoki Mavinga and Rosa Pardo.

Optimisation of a mixed Steklov Dirichlet Eigenvalue

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In this talk, I am going to talk about an eigenvalue optimisation problem over a family of doubly connected domains $U := D \setminus \Omega$ in $\mathbb{R}^2$ where ∂D, one boundary component, is a circle while the other component, $\partial\Omega$, enjoys a dihedral symmetry. The Boundary Value Problem under consideration is $\Delta u = 0$ on $D \setminus \Omega$, $u = 0$ on ∂D and $\frac{\partial u}{\partial n} = \sigma u$ on $\partial\Omega$. We study the behaviour of the first nonzero eigenvalue of this problem as the domain Ω rotates about its own center by an angle θ in the anticlockwise direction. We also investigate if there is any symmetry, monotonicity in the behaviour of the eigenvalue as a function of θ, and try to find global maximisers and global minimisers of the eigenvalue with respect to θ. This is based on a joint work with Sagar Basak, Ravi Prakash and Sheela Verma.

Serrin's problem revisited: the case of the generalized Monge-Ampère equations

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This talk deals with some Serrin's type symmetry results for some Monge-Ampère type problems with spherical or ellipsoidal free boundaries. First of all, we give a short review of a classical symmetry result in potential theory, due to J. Serrin [Arch. Rational Mech. Anal. 43 (1971)], presenting also some interesting alternative proofs of his result. Thereafter, we present some of our recent results in this direction, obtained for different classes of fully nonlinear elliptic problems, mostly of Monge-Ampere type. We'll thus show that some of our free boundary problems are solvable if any only if the free boundary is spherical or ellipsoidal.

The talk is based on a joint work with Prof. G. Porru [1].

References

[1] C. Enache, G. Porru, *Problems for generalized Monge–Ampȓe equations*, Canadian Math, Bull., 67(2) (2024), 265-278.

L^2-stability of the Heisenberg Uncertainty Principle on the hyperbolic space

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This talk is devoted to the study of weighted Poincaré inequalities on hyperbolic space, where the weight functions depend on a scaling parameter. This leads to a new family of scale-dependent Poincaré inequalities with Gaussian type measure on the hyperbolic space. As a result, we derive both scale-dependent and scale-invariant L^2-stability results for the Heisenberg uncertainty principle.

Modified Schrödinger Equations Involving Stein-Weiss Type Exponential Critical Nonlinearity

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In this talk, we will discuss the existence results of quasilinear/modified Schrödinger equation involving critical Choquard type exponential nonlinearity in bounded domain as well as in unbounded domain. In particular, we will discuss the existence results of following quasilinear Schrödinger equation

$$-\Delta_N u - \Delta_N(u^2)u + V(x)|u|^{N-2}u = \left(\int_{\mathbb{R}^N} \frac{F(y,u)}{|y|^{\beta}|x-y|^{\mu}}\, dy\right)\frac{f(x,u)}{|x|^{\beta}} \quad \text{in } \mathbb{R}^N,$$

where $N \geq 2$, $0 < \mu < N$. The potential $V : \mathbb{R}^N \to \mathbb{R}$ is a continuous function satisfying $0 < V_0 \leq V(x)$ for all $x \in \mathbb{R}^N$ and some suitable assumptions. The nonlinearity $f : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}$ is a continuous function with critical exponential growth in the sense of the Trudinger-Moser inequality and $F(x,s) = \int_0^s f(x,t)dt$ is the primitive of f.

The limiting case of the fractional Caffarelli-Kohn-Nirenberg inequality in dimension one

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The fractional Caffarelli-Kohn-Nirenberg (CKN) inequality in dimension $n = 1$ states that in the following range of parameters

$$\gamma \in (0, \frac{1}{2}), \quad \alpha \leq \beta \leq \alpha + \gamma, \quad -2\gamma < \alpha < \frac{1-2\gamma}{2},$$

we have

$$\Lambda \left(\int_{\mathbb{R}} \frac{|u(x)|^p}{|x|^{\beta p}} \, dx \right)^{\frac{2}{p}} \leq \int_{\mathbb{R}} \int_{\mathbb{R}} \frac{(u(x) - u(y))^2}{|x-y|^{n+2\gamma} |x|^{\alpha} |y|^{\alpha}} \, dy \, dx \tag{1}$$

for every $u \in C_c^\infty(\mathbb{R})$, provided

$$p = \frac{2}{1 - 2\gamma + 2(\beta - \alpha)}.$$

We also recall the following Moser-Trudinger-Onofri type inequality on $\mathbb{R}$: for every $v \in C_c^\infty(\mathbb{R})$

$$\log \frac{1}{2\pi} \int_{\mathbb{R}} e^v \, d\mu \leq \frac{1}{4\pi} \int_{\mathbb{R}} v(-\Delta)^{\frac{1}{2}} v \, dx + \frac{1}{2\pi} \int_{\mathbb{R}} v \, d\mu, \quad d\mu := \frac{2}{1+x^2} dx \tag{2}$$

In this talk we will show how to derive (2) from (1) by suitably choosing the parameters $\alpha, \beta \to 0$ and $\gamma \uparrow \frac{1}{2}$. Our proof is based on a priori estimates for the optimizers of (1), and the classification of solutions to a singular Liouville equation on $\mathbb{R}$.

Semilinear elliptic equations with critical nonlinearity: effect of perturbation

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The role played by lower order terms on the existence of solutions for semilinear elliptic problems with critical nonlinearity like the Yamabe problem, Brezis Nirenberg problem etc has been a topic of intense discussion in the past many decades. In this talk we will discuss a semilinear elliptic problem with Hardy-Sobolev type critical nonlinearity and the effect of nonlinear perturbations for the existence of lower energy solutions. This is a joint work with Sarika Goyal.

Shape sensitivity analysis for a liquid crystal model

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We shall discuss a shape sensitivity result, obtained in collaboration with Luis González (UACH) [2], for the equilibrium energy, in the Oseen-Frank model, of a nematic liquid crystal under normal anchoring conditions on the boundary. As an application we show that, for a nematic liquid crystal confined between two parallel cylindrical domains, the coaxial configuration is a critical state for the equilibrium energy, partially resolving a conjecture made by Alouges and Coleman [1].

References

[1] Alouges, F., Coleman, B.D.: Numerical bifurcation of equilibria of nematic crystals between non-co-axial cylinders. *Mathematical Models and Methods in Applied Sciences*, 11(3):459–473, 2001.

[2] González-Valencia, L., Mahadevan, R.: Shape Sensitivity for a Class of Liquid Crystal Models *J. Optim Theory Appl.* **204**, 52 (2025). https://doi.org/10.1007/s10957-025-02614-7

NonLinear Liouville Theorem involving pucci extremal operator.

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Abstract

In this talk, we establish a nonlinear Liouville theorem for equations of the form

$$-\mathcal{M}^{+}_{\lambda,\Lambda}(D^2u) = h(x_1)f(u), \quad u \geq 0 \quad \text{in} \quad \mathbb{R}^N, \quad \sup_{\mathbb{R}^N} u < +\infty,$$

where $\mathcal{M}^{+}_{\lambda,\Lambda}$ denotes the Pucci extremal operator, and $h \in C^{\alpha}(\mathbb{R}) \cap C^1(\mathbb{R})$ for some $\alpha \in (0,1)$ is an increasing function with $h(0) = 0$. The function f is nonnegative. We establish a Liouville-type theorem and use it to derive a priori estimates for positive solutions of indefinite superlinear pucci extremal equations.

Excess decay for quasilinear equations in Heisenberg Groups

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Abstract: The gradient of p-harmonic functions in Euclidean spaces satisfy a nice excess decay estimate which usually implies that these functions are $C^{1,\alpha}$ regular–a fact that is nontrivial to prove in the case of $p \neq 2$ due to the non-linearity of the p-Laplace equations. For solutions of homogeneous p-subLaplace equations in Heisenberg groups, proving that the horizontal gradient are Hölder continuous is much more delicate, and it took more than two decades to obtain a complete understanding. This is mainly due the existence of the nontrivial commutator. For the same reason, it was conjectured that the horizontal gradient could not enjoy any Euclidean like excess decay estimates. In this talk, I will present some results showing that the horizontal gradient indeed satisfies Euclidean like excess decay estimates, disproving the conjecture. I will also present a variant of the nonlinear Stein theorem in Heisenberg Groups setting, established using these excess decay estimates.

Stability of the Pohozaev obstruction

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In this talk, we will consider the issue of the non-existence of solutions to Brezis-Nirenberg type equations on bounded Euclidean domains (dimension ≥ 3). The leading order terms of this equation are invariant under conformal transformations which leads to the classical Pohozaev identity, and this in turn shows the non-existence of solutions to the equation when the domain is star-shaped. We will give a brief description of the persistence of this non-existence phenomenon under perturbations. The proofs and techniques are based on sharp blow-up analysis.

On the moving plane method in the Heisenberg groups

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I shall discuss the adaptation of the moving plane method in the Heisenberg groups and discuss its applications to prove symmetry of solutions in bounded and unbounded domains. Particularly, I will mention the classification of solution of the CR Yamabe problem and the Gidas-Ni-Nirenberg type of result in the Heisenberg group.

On some critical cases of local and fractional boundary Hardy inequality

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Boundary Hardy inequality states that if $1 < p < \infty$ and Ω is a bounded Lipschitz domain in $\mathbb{R}^d$, then

$$\int_\Omega \frac{|u(x)|^p}{\delta_\Omega^p(x)} dx \leq C \int_\Omega |\nabla u(x)|^p dx, \quad \forall\, u \in C_c^\infty(\Omega) \tag{1}$$

where $\delta_\Omega(x)$ is the distance function of the point $x \in \Omega$ to $\partial\Omega$. B. Dyda generalised the above inequality to the fractional setting, which says, for $sp > 1$

$$\int_\Omega \frac{|u(x)|^p}{\delta_\Omega^{sp}(x)} dx \leq C \int_{\Omega\times\Omega} \frac{|u(x) - u(y)|^p}{|x-y|^{d+sp}} dxdy, \quad \forall\, u \in C_c^\infty(\Omega) \tag{2}$$

The first and the second inequality is not true for $p = 1$ and $sp = 1$ respectively. In this talk, we will present the appropriate inequalities for the critical cases: $p = 1$ for (1) and $sp = 1$ for the second inequality (2). We will also discuss the case when the weight function δ_Ω in the second inequality is replaced by distance function from a k–dimensional sub manifold of Ω and some related applications to fractional Moser-Trudinger inequality.

Sublinear positone and semipositone problems on the exterior of a ball in $\mathbb{R}^2$

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We will discuss the study of positive solutions to problems of the form,

$$\begin{cases} -\Delta u = \lambda K(x) f(u) & \text{in } B_1^c, \\ u(x) = 0 & \text{on } \partial B_1, \end{cases} \tag{1}$$

where $B_1^c = \{x \in \mathbb{R}^2 : |x| > 1\}$, λ is a positive parameter, $K : B_1^c \to \mathbb{R}^+$ belongs to a class of Hölder continuous functions which satisfy certain decay assumptions and $f : (0, \infty) \to \mathbb{R}$ belongs to a class of Hölder continuous functions which are sublinear. For a class of positone problems of the form (1), we will discuss the existence of multiple positive solutions for a range of the parameter λ and the uniqueness of positive solutions for either sufficiently large or small values of λ. For a semipositone problem of the form (1), we will discuss an existence result for large values of λ. Our results extend the study of similar problems on exterior domains in $\mathbb{R}^n$, $n > 2$.

The talk is based on the work [1].

References

[1] Anumol Joseph, Lakshmi Sankar. Sublinear positone and semipositone problems on the exterior of a ball in $\mathbb{R}^2$. Under Review.

Homogeization of semi-linear parabolic optimal contol problem in domain with oscillating boundary.

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We consider an optimal control problem governed by a semi-linear parabolic equation in a two-dimensional domain Ω_ϵ with oscillating boundary. The state equation and cost function involve highly oscillatory periodic coefficients, A^ϵ and B^ϵ. Our goal is to analyze the limiting behavior (as $\epsilon \to 0$)of the optimal control and state as the oscillations become finer. Additionally, we identify the homogenized optimal control problem and establish a corrector result for the state variable.

The talk is based on the work [1].

References

[1] Nandakumaran, A.K., Raj, R. & Sardar, B. C. Optimal control and homogenization of semi-linear parabolic problem with highly oscillatory coefficients in an oscillating domain, Nonlinear Differential Equations and Applications NoDEA (In press).

On sharpened singular Adams' type inequalities and applications to a fourth-order equation

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In this talk, we discuss a sharp version of Adams' type inequality in a suitable higher-order function space with singular weight in $\mathbb{R}^n$. In addition, we also provide proof of a sharp singular concentration-compactness principle due to Lions' as an improvement of this singular Adams' inequality. We shall demonstrate a new compact embedding, which plays a crucial role in our arguments. Moreover, as an application of these results, by employing the mountain pass theorem, we study the existence of nontrivial solutions to a class of nonhomogeneous quasilinear elliptic equations involving $(p, \frac{n}{2})$-biharmonic operator with singular exponential growth as follows

$$\Delta_p^2 u + \Delta_{\frac{n}{2}}^2 u = \frac{g(x,u)}{|x|^\gamma} \quad \text{in} \quad \mathbb{R}^n,$$

with $n \geq 4$, $1 < p < \frac{n}{2}$, $\gamma \in (0, n)$ and the nonlinear term $g : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}$ is a Carathéodory function, which behaves like $\exp(\alpha |s|^{\frac{n}{n-2}})$ as $|s| \to +\infty$ for some $\alpha > 0$. This talk is based on the work [1].

References

[1] Deepak Kumar Mahanta, Tuhina Mukherjee, and Abhishek Sarkar. On sharpened singular Adams' type inequalities and applications to $(p, \frac{n}{2})$-biharmonic equations, 2024. https://doi.org/10.48550/arXiv.2412.11176

Optimal harvesting for a logistic model with grazing

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We consider semi-linear elliptic equations of the following form:

$$\begin{cases} -\Delta u = \lambda[u - \dfrac{u^2}{K} - c\dfrac{u^2}{1+u^2} - h(x)u] & \text{in } \Omega, \\ \dfrac{\partial u}{\partial \eta} + qu = 0 & \text{on } \partial\Omega, \end{cases}$$

where, $h \in U = \{h \in L^2(\Omega) : 0 \leq h(x) \leq H\}$. We prove the existence and uniqueness of the positive solution for large λ. Further, we establish the existence of an optimal control $h \in U$ that maximizes the functional $J(h) = \int_\Omega h(x)u_h(x)\, \mathrm{d}x - \int_\Omega (B_1 + B_2 h(x))h(x)\, \mathrm{d}x$ over U, where u_h is the unique positive solution of the above problem associated with h, $B_1 > 0$ is the cost per unit effort when the level of effort is low and $B_2 > 0$ represents the rate at which the cost rises as more labor is employed. Finally, we provide a unique optimality system.

The talk is based on the work [1].

References

[1] Ardra A., Mohan Mallick and Sarath Sasi, Optimal harvesting for a logistic model with grazing. *Evol. Equ. Control Theory*, 14, No. 3, 511-529 (2025).

Uniqueness of positive solutions for a class of nonlinear elliptic equations with Robin boundary conditions

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We prove uniqueness of positive solutions to the BVP

$$\begin{cases} -\Delta u = \lambda f(u) & \text{in } \Omega, \\ \frac{\partial u}{\partial n} + bu = 0 & \text{on } \partial\Omega, \end{cases}$$

when the parameter λ is large independent of $b \in (0, \infty)$. Here Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary $\partial\Omega$, $f : [0, \infty) \to [0, \infty)$ is continuous, concave for u large, and sublinear at ∞. (Joint work with D.D. Hai & Xiao Wang.)

Two positive solutions to a semilinear spectral problem with various convexity nonlinearities

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Population models with a diffusive spread of a population following some kind of nonlinear growth pattern have a long history. Selecting, for the growth, a mixture of a concave nonlinearity at low population densities and a convex one at high densities, in the late 1970s and early 1980s a number of researchers have developed powerful methods of partial differential equations and functional analysis to treat such problems assuming a uniform, space--idependent growth throughout the domain of the habitat. A typical result was the possibility of two positive equilibria of a lower and a higher population density, pointwise ordered throughout the habitat. In our presentation we will construct a similar pair of equilibria in a model with space-dependent growth: concave in one subdomain and convex in the other one, linear on the boundary (or region) between the two subdomains.

We will discuss the question of ***existence*** and ***multiplicity*** of *positive solutions* to the semilinear elliptic Dirichlet problem

$$- \Delta u = \lambda \, u(x)^{q(x)-1} \quad \text{for } x \in \Omega \, ; \qquad u = 0 \quad \text{on } \partial\Omega \, , \tag{1}$$

where $\Omega \subset \mathbb{R}^N$ is a bounded domain with the boundary of class $C^{1,\alpha}$, $\lambda \in \mathbb{R}$ a spectral parameter, and $f(x,u) = |u|^{q(x)-1} \, u$ is a **signed $q(x)$-power** of the unknown function of (a positive variable) $u \in (0, \infty)$ which depends on the point $x \in \Omega$.

We will briefly present basic methods for treating the semilinear elliptic Dirichlet problem

$$\Delta u + \lambda \, u(x)^{q(x)-1} = 0 \tag{2}$$

with a ***convex*** and ***concave*** nonlinear reaction

$$f(x, \cdot \,) \colon s \longmapsto |s|^{q(x)-2} s \colon \mathbb{R}_+ \subset \mathbb{R} \to \mathbb{R} \tag{3}$$

which (for $s \geq 0$) is convex in an nonempty open subset

$$\Omega_+ \stackrel{\text{def}}{=} \{x \in \Omega \colon q(x) > 2\} \tag{4}$$

and concave in another nonempty open subset

$$\Omega_- \stackrel{\text{def}}{=} \{x \in \Omega \colon q(x) < 2\} \tag{5}$$

of a bounded domain $\Omega \subset \mathbb{R}^N$. Here, $\lambda \in \mathbb{R}_+$ is a nonnegative spectral parameter which decides about the existence and multiplicity of positive weak solutions (at least two). For $0 < \lambda \leq \lambda^*$ we obtain a "stable" C^1-solution $u \equiv u_\lambda$ by monotone iterations. For $0 < \lambda < \lambda^*$ we obtain another C^1-solution, v, by the LERAY–SCHAUDER degree theory, that seems to be "unstable" and satisfies $v(x) > u_\lambda(x)$ for all $x \in \Omega$. Our main tools are fixed points of concave (subhomogeneous) mappings that take advantage of $\Omega_- \neq \emptyset$ in contrast to a priori estimates for "large" positive solutions, v, taking advantage of $\Omega_+ \neq \emptyset$. Our a priori estimates are obtained by a somewhat intriguing application of Young's inequality where Hölder's inequality does not seem to be fine enough. Our main contribution is a method how to handle the interplay between *convex* and *concave* nonlinearities in two disjoint nonempty open subsets of a domain Ω (connected in $\mathbb{R}^N$), as opposed to the classical works assuming a nonlinearity $f(s)$ being *concave* for small values of $s \in \mathbb{R}_+$ and *convex* for large $s \in \mathbb{R}_+$, uniformly in Ω.

The Dirichlet Laplace operator being linear, we observe that the elliptic equation in problem (1) is ***convex*** (***concave***, respectively) at a given point $x \in \Omega$, provided $q(x) \geq 2$ $(1 < q(x) \leq 2)$. This is a typical problem with variable powers (exponents).

Finally, if time permits, we will discuss also the classical question of ***uniqueness*** for a related problem with the $p(x)$-Laplacian provided $q(x) \leq \mathrm{const}_1 < \mathrm{const}_2 \leq p(x)$ holds for all $x \in \Omega$.

References

[1] P. TAKÁČ and J. GIACOMONI, *A $p(x)$-Laplacian extension of the Díaz--Saa inequality and some applications*, Proc. Roy. Soc. Edinburgh, Sect. **A 150**(1) (2020), 205–232.

Optimal geometric inequalities and proximity to geodesic spheres

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First, I will review some classical geometric inequalities for hypersurfaces in Euclidean space, where the equality case characterizes geodesic spheres, such as Reilly's upper bound for the first eigenvalue of the Laplace operator. After observing that these inequalities arise from a lower bound on the L^2-norm of the position vector, I will present a stability result for this lower bound.

The talk is based on the work [1].

References

[1] Roth, J., Upadhyay, A., New eigenvalue pinching results for Euclidean domains, Annali di Matematica Pura ed Applicata, vol 204, 307-326, (2025).

Steklov eigenvalues on doubly connected domains

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On concentric annular domains, Steklov eigenvalues behave differently when compared with mixed Steklov Dirichlet eigenvalues and mixed Steklov Neumann eigenvalues. In this talk, we will discuss about this difference and explore the Steklov eigenvalues on concentric annular domains. We will also give an isoperimetric bound for higher Steklov eigenvalues on certain symmetric doubly connected domains.

This talk is based on the work [1].

References

[1] Basak, Sagar and Verma, Sheela, *Bounds for higher Steklov and mixed Steklov Neumann eigenvalues on domains with holes, arXiv preprint arXiv:2412.17124*.

Flash Talks

On the first eigenvalue of a nonlinear Schrödinger type p-Laplace equation

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We consider an eigenvalue problem for the generalized Schrödinger type operator with Robin boundary condition given below.

$$\begin{cases} -\Delta_p u + V(x)|u|^{p-2}u = \lambda |u|^{p-2}u \quad \text{in } \Omega, \\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \eta} + \beta |u|^{p-2}u = 0 \quad \text{on } \partial\Omega, \end{cases}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with smooth boundary, $V \in C^1(\mathbb{R}^n)$, η denotes the outward unit normal, and β is a positive real constant. We study the properties of its first eigenvalue with respect to the potential, the boundary parameter β as well as the domain. First, we establish some properties of the smallest eigenvalue $\lambda_1(V)$ with respect to the potential. We then investigate the monotonicity of $\lambda_1(V)$ with respect to the Robin boundary parameter β by finding its derivative with respect to β. We also obtain a shape derivative formula for $\lambda_1(V)$ and use it to study domain monotonicity properties. We also study the above eigenvalue problem for a more general class of potentials $V \in L^q(\Omega)$, for $q > \frac{n}{p}$. For such V, we show the existence of a weak solution and establish the existence of potentials optimizing the lowest eigenvalue $\lambda_1(V)$.

Steklov eigenvalues on concentric annular domain

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In this talk, I will discuss the Steklov eigenvalue problem, focusing on concentric annular domain. While the general increasing sequence of Steklov eigenvalues for this domain remains an open question, I will present explicit expressions for the first and second smallest Steklov eigenvalues on this domain.

Some isoperimetric-type inequalities to the non-local composite membrane problem

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In this flash talk, we present a few isoperimetric-type inequalities associated with the first eigenvalue of the non-local composite membrane problem. We begin by discussing an analogue of the classical Faber–Krahn inequality in this non-local setting. Following this, we explore an isoperimetric inequality for the first eigenvalue of the non-local composite membrane problem on the intersection of two domains—an extension of a problem originally studied by Lieb for the Laplacian.

This talk will be based on the article [1].

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Normalized Solutions to the Kirchhoff-Choquard Equations with Combined Growth

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In this talk, we discuss the following equation:

$$-\left(a+b\|\nabla u\|_2^{2(\theta-1)}\right)\Delta u=\lambda u+\alpha(I_\mu*|u|^q)|u|^{q-2}u+(I_\mu*|u|^p)|u|^{p-2}u \text{ in } \mathbb{R}^N,$$

with the prescribed norm $\int_{\mathbb{R}^N}|u|^2=c^2$, where $N\geq 3$, $0<\mu<N$, $a,b,c>0$, $1<\theta<\frac{2N-\mu}{N-2}$, $\frac{2N-\mu}{N}<q<p\leq\frac{2N-\mu}{N-2}$, $\alpha>0$ is a suitably small real parameter, $\lambda\in\mathbb{R}$ is the unknown parameter which appears as the Lagrange's multiplier and I_μ is the Riesz potential. We establish existence and multiplicity results and further demonstrate the existence of ground state solutions under the suitable range of α. We demonstrate the existence of solution in the case of q is L^2-supercritical and $p=\frac{2N-\mu}{N-2}$, which is not investigated in the literature till now. In addition, we present certain asymptotic properties of the solutions. To establish the existence results, we rely on variational methods, with a particular focus on the mountain pass theorem, the min-max principle, and Ekeland's variational principle.

Tuning planar transverse domain wall dynamics in bilayer nanostructures using transverse magnetic fields, Rashba, and spin-Hall effects

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This work explores the tunability of a planar transverse domain wall with an arbitrary azimuthal angle, achieved through the application of a transverse magnetic field of adjustable strength and fixed orientation. We investigate the dynamics within a bilayer nanostructure comprising a ferromagnetic layer and a non-magnetic heavy metal layer, employing the Landau-Lifshitz-Gilbert equation as our theoretical framework.

The domain wall dynamics are analyzed using the collective coordinate method and a regular perturbation asymptotic approach, incorporating the combined influences of axial and transverse magnetic fields, spin-polarized electric currents, the Rashba effect, and the spin-Hall effect. Our study provides a comprehensive characterization of the domain wall profile, defined by sharply delineated domain boundaries and a precisely controlled transverse magnetic field distribution. Additionally, we detail the linear polar angle variation within the domain wall region, the tunability of the domain wall width, and the enhancement of domain wall velocity in the steady-state regime.

This talk is based on the work [1].

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Global existence and finite time blow-up of weak solutions for parabolic-elliptic Keller-Segel systems with flux dependent chemotactic coefficient

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We consider the following Keller-Segel system with chemotactic coefficient

$$\begin{cases} u_t = \triangle u - \chi \nabla \cdot (u f(|\nabla v|)\nabla v), \\ 0 = \triangle v - h(v) + g(u), \end{cases} \tag{1}$$

in a smooth bounded domain $\Omega \subset \mathbb{R}^n, n \geq 2$ with $f(\xi) = \big(\xi^{p-2}(1 + \xi^p)^{\frac{q-p}{p}}\big), 1 < q \leq p < \infty$. We show the existence of a globally bounded weak solution in L^∞ norm of (1), with $h(\xi) = \xi$, and $g(\xi) = \frac{\xi}{(1+\xi)^{1-\beta}}, \beta \in [0,1]$, provided $1 < q < \frac{n}{n-1}$.
Further, in radially symmetric domain $\Omega = B_R(0) \subset \mathbb{R}^n (n \geq 3)$, with $h(\xi) = M > 0, g(\xi) = \xi$, we assert that under a condition on the initial data, radial weak solutions blow-up in finite time when

$$\frac{n}{n-1} < q < 2,\ p \geq 2.$$

The proof involves with construction of a system of regularized problems dependent on $\epsilon \in (0,1)$ to make the coefficients sufficiently regular such that there exists a classical solution for every $\epsilon \in (0,1)$ and use of elliptic regularity theory, parabolic regularity theory, imbedding theorems and compactness arguments to validate the existence of a weak solution.

The talk is based on the work [1], [2].

References

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Multiplicity results for mixed local nonlocal equations with indefinite concave-convex type nonlinearity

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In this talk, we discuss the multiplicity of non-negative solutions for a class of equations involving mixed local-nonlocal nonhomogeneous operator. The problem features indefinite concave-convex nonlinearities with sign-changing weights and a parameter λ. The nonlinearity combines sublinear and superlinear growth terms, which can be either subcritical or critical. Our analysis is based on the study of fibering maps and the minimization of the associated energy functional over appropriate subsets of the Nehari manifold. For a specific operator of such kind, we establish a nonexistence result for sufficiently large λ in the subcritical case. The proof relies on a generalized eigenvalue problem for mixed local-nonlocal operators, whose key properties we develop as part of our analysis.

The talk is based on a joint work with Dr. R. Dhanya of IISER Thiruvananthapuram, and Dr. Jacques Giacomoni of Universite de Pau.

Reverse Faber-Krahn inequalities for the Logarithmic potential operator

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This talk focuses on the largest eigenvalue, $\tau_1(\Omega)$, of the Logarithmic potential operator $\mathcal{L}$ for a bounded open set $\Omega \subset \mathbb{R}^2$. For $\operatorname{diam}(\Omega) \leq 1$, reverse Faber-Krahn type inequalities for $\tau_1(\Omega)$ are established under polarization and Schwarz symmetrization. These results are then applied to analyze the monotonicity of $\tau_1(\Omega \setminus \mathcal{O})$ with respect to certain translations and rotations of an obstacle $\mathcal{O}$ within Ω. In addition, we present the smallest eigenvalue $\tilde{\tau}_1(\Omega)$ of $\mathcal{L}$ for domains with transfinite diameter greater than 1. Characterization of the eigenvalues of $\mathcal{L}$ on B_R, including $\tilde{\tau}_1(B_R)$ will also be discussed.

The talk is based on the work [1].

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Nonlocal problem with critical exponential nonlinearity of convolution type: a non-resonant case

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In this paper, we study the following class of weighted Choquard equations

$$-\Delta u = \lambda u + \left(\int\limits_{\Omega} \frac{Q(|y|)F(u(y))}{|x-y|^{\mu}} dy \right) Q(|x|)f(u) \quad \text{in} \quad \Omega \quad \text{and} \quad u = 0 \quad \text{on} \quad \partial\Omega,$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with smooth boundary, $\mu \in (0, 2)$ and $\lambda > 0$ is a parameter. We assume that f is a real-valued continuous function satisfying critical exponential growth in the Trudinger-Moser sense, and F is the primitive of f. Let Q be a positive real-valued continuous weight, which can be singular at zero. Our main goal is to prove the existence of a nontrivial solution for all parameter values except when λ coincides with any of the eigenvalues of the operator $(-\Delta, H_0^1(\Omega))$.

References

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Polarization and Shape Variations

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Polarization is a simplest symmetrization of functions on $\mathbb{R}^N$. In this talk, we discuss a few applications of polarization to some shape variation problems in $\mathbb{R}^N$.

Differential inclusions for differential forms

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We will study the existence of $u \in W_0^{1,\infty}\left(\Omega; \Lambda^k(\mathbb{R}^n)\right)$ satisfying the following differential inclusion problem:

$$\mathrm{d}\, u \in E \text{ a.e. in } \Omega,$$

where $k, n \in \mathbb{N}$ with $0 \leq k \leq n-1$, $\Omega \subseteq \mathbb{R}^n$ is open, bounded and $E \subseteq \Lambda^{k+1}(\mathbb{R}^n) \setminus \{0\}$ is a given set. We will also talk on the application of differential inclusions on variational problems.

References

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Existence results for degenerate elliptic equations involving a Hardy potential and nonlinear singularity

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In recent years, there has been growing interest in nonlinear singular elliptic partial differential equations, both for pure mathematical research and due to their real-world applications. In this work, we investigate the existence and regularity of nonnegative solutions to a class of nonlinear degenerate singular elliptic equations involving a Hardy potential in the lower-order terms. Specifically, we consider the following problem on an open bounded domain $\Omega \subset \mathbb{R}^N$, $N \geq 3$:

$$\begin{cases} -\mathrm{div}\left(\dfrac{|\nabla u|^{p-2}\nabla u}{(1+u)^{(p-1)\tau}}\right) = \mu \dfrac{u^s}{|x|^p} + \dfrac{f}{u^\sigma},\ u \geq 0 \quad \text{in } \Omega, \\ \qquad\qquad\qquad\qquad u = 0 \quad \text{on } \partial\Omega. \end{cases}$$

Here μ, σ, τ and s are positive real numbers, and f is a nonnegative function that belongs to a suitable Lebesgue space. Despite the singular nature of the right-hand side, we demonstrate that it has a regularizing effect on the solutions. To establish our results, we approximate the problem by truncating the singular term to ensure it is well-defined at the origin. Then, using a priori estimates, we pass to the limit in the approximated problems to obtain a solution.

A Hopf-type strong comparison principle for singular problems with mixed local-nonlocal operators

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We present a Hopf-type strong comparison principle for singular problems involving mixed local-nonlocal operators. Suppose u_1 and u_2 are two positive $C^{1,\alpha}(\overline{\Omega})$-functions satisfying

$$\begin{aligned} -\Delta_p u_1 + (-\Delta_q)^s u_1 - \frac{1}{u_1^\beta} &= f_1 \text{ in } \Omega \\ -\Delta_p u_2 + (-\Delta_q)^s u_2 - \frac{1}{u_2^\beta} &= f_2 \text{ in } \Omega \\ u_1 = u_2 &= 0 \text{ in } \mathbb{R}^N \setminus \Omega \end{aligned}$$

where $0 < \beta < 1$, Ω is a smooth, bounded domain in $\mathbb{R}^N$, f_1, f_2 are non-negative continuous functions and $f_1 \leq f_2$ in Ω. We show that if $u_1 \not\equiv u_2$ then $u_2 - u_1 \geq cd(x)$ in Ω, or in other words, $\frac{\partial u_2}{\partial \nu} < \frac{\partial u_1}{\partial \nu}$ on $\partial\Omega$. The result has applications in establishing several multiplicity results.

The talk is based on the work [1].

References

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Spike layered solutions for a singularly perturbed elliptic system on Riemannian manifold

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Let (M, g) be a connected compact smooth Riemannian manifold of dimension $N \geq 3$ without boundary. We consider the singularly perturbed elliptic system

$$\begin{cases} -\varepsilon^2 \Delta_g u + u = |v|^{q-1} v, & \\ -\varepsilon^2 \Delta_g v + v = |u|^{p-1} u & \text{in } M, \\ \qquad u, v > 0 & \text{in } M, \end{cases} \tag{P_ε}$$

where Δ_g is the Laplace-Beltrami operator on M, $\varepsilon > 0$ is the perturbation parameter. The exponents p, q satisfy $p, q > 1$ and

$$\frac{1}{p+1} + \frac{1}{q+1} > \frac{N-2}{N}. \tag{HC}$$

In this talk, we will discuss the existence and asymptotic behaviour of the solution for (P_ε) as $\varepsilon \to 0$. Using a dual variational method, we prove the existence of a mountain pass solution $(u_\varepsilon, v_\varepsilon)$. We will also show that $(u_\varepsilon, v_\varepsilon)$ attains its maximum at a unique and common point p_ε which converges to a maximum point of the scalar curvature on M as $\varepsilon \to 0$.

Fractional Orlicz-Hardy inequalities

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We establish fractional Orlicz-Hardy inequalities for a Young function that satisfies the doubling condition. Further, we identify the critical cases for each Young function and prove fractional Orlicz-Hardy inequalities with logarithmic correction.

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Critical Choquard-Kirchhoff type equation involving Hardy Potential

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We study the following equation

$$\begin{aligned} -\left(a+b\left(\int_{\mathbb{R}^N}|\nabla u|^2dx\right)^{\theta-1}\right)\Delta u - a\mu\frac{u}{|x|^2} &= \lambda g(x)|u(x)|^{q-2}u(x) \\ &+\gamma\left(\int_{\mathbb{R}^N}\frac{|u(y)|^{2^*_\alpha}}{|x-y|^\alpha}dy\right)|u(x)|^{2^*_\alpha-2}u(x) \quad \text{in } \mathbb{R}^N\setminus\{0\}, \end{aligned}$$

where $N \geq 3$ with $a \geq 0$, $b > 0$, $\theta \geq 1$, $\mu \in \left[0, \frac{(N-2)^2}{4}\right)$, $0 < \alpha < N$, $2^*_\alpha = \frac{2N-\alpha}{N-2}$ is the critical exponent in the sense of Hardy-Littlewood-Sobolev inequality and $\lambda > 0$, $\gamma > 0$ are parameter. The function $g \in L^r(\mathbb{R}^N)$, where $r = \frac{2^*}{2^*-q}$ if $1 \leq q < 2^*$ and $r = \infty$ if $q = 2^*$. We proves the existence of infinitely many solutions to the above equation for different range of q.

To proof the theorems we use concentration compactness lemma, symmetric mountain pass lemma, Z_2 symmetric version of mountain pass theorem due to Kajikiya .

The talk is based on the work [1, 2, 3] etc.

References

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On the second Steklov eigenvalue on trees

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Let G be a graph with boundary B. We define the discrete Steklov operator $\Lambda : R^B \to R^B$ by

$$\Lambda(f) = \frac{\partial \hat{f}}{\partial n},$$

where $\hat{f}$ is the harmonic extension of f to the whole graph V, and the discrete normal derivative is given by

$$\frac{\partial \hat{f}}{\partial n}(x) = \sum_{(x,y)\in E} (f(x) - f(y)).$$

In this problem, first I discuss the upper bound of the second Steklov eigenvalue on trees.

Next, I will discuss explicit form of the second eigenvalue, which depends on the number of vertices and the number of boundary vertices.